Universal Gravitation

Physics 203, Profs. Bean & Yaverbaum
John Jay College of Criminal Justice, the CUNY

Read chapter 13, sections 13-1, 13-2, and 13-3 in the textbook.
(13-1 is the most important for our purposes, but the other two are good to know about.)

Recall the universal gravitational constant: $G = 6.67 \times 10^{-11}\ \mathrm{N} \cdot \frac{\mathrm{m}^2}{\mathrm{kg}^2} = 6.67 \times 10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{s}^2}$

Recall Netwon's Law of Universal Gravitation: $F_G = G\frac{m_1 m_2}{r^2}$

I. Warm Up

A large 900,000 kg asteroid and a small 10,000 kg asteroid are floating in space. The centers of the asteroids are 200 meters apart.

A. What is the force that the big asteroid exerts on the little asteroid?
B. What is the force that the little asteroid exerts on the big asteroid?

A scientist watching from a distant planet sees the asteroids accelerate towards the other.

C. What is the acceleration of the large asteroid? (Hint: use Newton's Second Law))
D. What is the acceleration of the small asteroid?
E. What is the acceleration of the small asteroid *relative to the large asteroid*?

II. Baby Space Dive

Professor Yaverbaum jumps out of a small orbital spaceship 10 kilometers above the surface of a small, unknown planet. He discovers that he begins falling with an acceleration of 15 $\mathrm{m/s^2}$. Based on telemetric data, Yaverbaum knows that the planet has a radius of 4,000 kilometers. Find the mass of the planet!

Step 1: How far is Yaverbaum from the center of the planet?
HINT: Look back at the given information. Everything you need is there.
Step 2: Apply Newton's 2^{nd} Law to Yaverbaum. (Assume air-resistance is negligible).
Step 3: Solve for the force of gravity on Yaverbaum. (Leave Yaverbaum's mass, M_y, as a ***variable*** in your answer.)
Step 4: Write down Newton's Universal Law of Gravitation. Plug in any values you know on both sides of the equation.
Step 5: Solve for the mass of the planet. (Note, these equations ***cannot*** be used to find Yaverbaum's mass.)

III. Orbit

A 20,000,000,000 kg moon is orbiting a 1,000,000,000,000 kg planet at a distance of 500 km from the planet's center.

A. Find the speed of the moon relative to the planet.
   Hint 1: Compute the force of gravity on the moon.
   Hint 2: Draw an FBD of the moon and include its orbital path as a dotted line.
   Hint 3: Apply Newton's Second Law to the radial axis.
   Hint 4: Find the centripetal acceleration of the moon.
   Hint 5: Find the speed of the moon.

B. Find the time it takes for the moon to complete one trip around the planet.
   Hint 1: You already have speed. You just need distance.
   Hint 2: You may want to convert your answer into hours… or into Earth Years. It should be several Earth Years.

C. A collision with an asteroid causes the moon to move closer to the planet. The period of its orbit is now only 50 Earth days. Find R, the new distance between (the centers of) the moon and the planet.
   Step 1: Using Newton's Law of Gravitation, express the force of gravity, in terms of R.
   Step 2: Write an expression for distance traveled in one orbit, in terms of R.
     Hint: What shape is it traveling in?
   Step 3: Write an expression for the moon's speed, in terms of R.
     Hint: Use your work from Step 1. You have distance & time. You want speed.
   Step 4: Using your work fr step 2, express centripetal acceleration, in terms of R.
   Step 5: Apply Newton's 2^{nd} Law to the Moon. Plug in what you know.
   Step 6: Solve for R. All other variables should drop out.

IV. Work Done By Gravitation

Consider the scenario described in problem II: Yaverbaum has jumped out of a spacecraft and is falling towards an unknown planet. In the warm-up, you found Yaverbaum's acceleration immediately after jumping out of his spaceship.

A. Explain why knowing this acceleration does NOT allow you to find out either how long Yaverbaum will take to fall a certain distance nor how fast he will be going after falling that distance.
B. Write a function that gives the magnitude of the gravitational force on Yaverbaum as a function of how far he is from the planet's center.
   HINT: Your function will be of the form F = ___x, where x is dist. from planet's center.
C. If Yaverbaum falls 1km, how much work does gravity do on him as he falls?
   HINT: Recall the definition of work when forces are non-constant. Plug in your function.
D. Assuming Yaverbaum fell from rest, compute his final velocity.
   HINT: W-KE theorem.