Impulse, Momentum, Collision

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Complete the textbook readings: 9-1, 9-2, 9-3, 9-4, 9-5, 9-6, 9-7, 9-8

(If you are in a hurry, you may skip the second part of 9-1, entitled "systems of particles." You may also skip the second part of 9-4, entitled "Series of Collisions." All the rest is required reading, though some of it may be familiar from class, allowing you to move more quickly. Also, note that the text deals with 3-dimensions, using unit vectors: î, ˆj, ˆk. You do ***not*** need to be familiar with this notation, ***nor*** will you need to work with three-dimensional problems in this course.)

Recall the special definition of impulse when forces are constant: $\vec{J} \equiv \vec{F}t$

Recall the general definition of impulse when forces are ***not*** constant: $\vec{J} \equiv \int_{t_0}^{t_f} \vec{F}dt$

Recall the definition of momentum: $\vec{p} \equiv m\vec{v}$

Recall the impulse-momentum theorem: $\vec{J} = \Delta\vec{p}$

(in other words, impulse is the ***transfer*** of momentum.)

I. Impulse-Momentum Theorem.

A. A hand pushes an 8 kg box across an essentially frictionless table. The hand pushes with a constant horizontal force of 15 Newtons. It does so for 3 seconds.

i. No matter what initial velocity it may have had, the box's ***linear momentum*** will change by a certain amount. What is this amount?

ii. If the box starts from rest, what final velocity will it achieve?

iii. If the box starts with an initial velocity of 5 m/s--pointing in the ***same*** direction as the push--then what final velocity will it achieve?

iv. If the box starts with an initial velocity of 5 m/s--pointing in the ***opposite*** direction as the push--then what final velocity will it achieve?

B. Wonder woman is pushing a horse through outer space. She pushes for a period of 10 seconds, and she gets stronger as she pushes. The force of her push at any given moment is equal to $100t^3$, where t is the time in seconds since she began pushing.

i. Compute the total impulse on the horse during these 10 seconds.

ii. Assume the horse starts out from rest. Compute the final velocity of the horse.

II. Conservation of linear momentum: A brief explanation.

A net external impulse exerted on a system transfers linear momentum either to or from that system. Hence, if NO NET EXTERNAL IMPULSE acts on a given system, then no linear momentum is transferred to nor from that system. Any system that is free from net external impulses, therefore, ***conserves linear momentum***.

The two central cases for linear momentum conservation are COLLISIONS and EXPLOSIONS (the latter being the former run in reverse time).

Collisions are commonly and easily pictured on a billiard table. At first, this may be surprising, because there seem to be all kinds of external frictional forces affecting pool balls on a billiard table. In fact, the effect of these frictional forces ***during the moment of collision*** is negligible (we'll discuss this more in class).

What is ***not*** negligible is the effect of the billiard balls ***on each other***. We observe that one ball exerts an impulse on the other. By Newton's third law, the second ball exerts an equal but opposite impulse right back on the first ball. The net impulse on the system is zero. In short:

Collisions conserve linear momentum.

Now picture an automobile crashing into another. Cars are far less "elastic" ("springy") than balls. When one car slams into another, the metal gets crushed, you hear a tremendous sound, pieces fly, and the immediate environment can get extremely hot. All of this means that mechanical energy is getting lost—converted into heat, sound, etc. ***But linear momentum is still conserved--even when energy is not***.

We can use conservation of linear momentum to solve problems involving collisions.

III. Pool Ball Meets Tennis Ball

A pool ball has a mass of .6 kg (600 grams) . At some moment, it has an instantaneous velocity of +3 m/s. A tennis ball has a mass .4 kg (400 grams). At some moment it has an instantaneous velocity of -3 m/s. The two balls experience a head-on collision.

a) If the tennis ball exits the collision at a instantaneous velocity of +1 m/s, with what instantaneous velocity does the pool ball exit the collision?

How to solve this? We know that momentum is conserved during a collision. In other words,

Total momentum right before the collision = Total momentum right after the collision.

And by "total momentum" we mean the total momentum of the two objects involved in the collision. In other words,

Total momentum = The momentum of the pool ball + The momentum of the tennis ball.

So, putting this all together, we have

$$M_1 V_{1i} + M_2 V_{2i} = M_1 V_{1f} + M_2 V_{2f}$$

It's easy to compute momentum if we know mass and velocity. And in fact we know a lot of masses and velocities. We know:

- the masses of both objects,
- the velocities of both objects before the collision, and
- the velocity of one of the objects after the collision.

And we're looking for the velocity of the other object after the collision.

So compute the ***total*** linear momentum before the collision and set it equal to the total linear momentum after the collision. You will have one variable in this equation: $V_{poolball\text{-}final}$.

b) If there was glue all over the balls so that they ***stuck together*** at impact, with what instantaneous velocity would the 2-ball system exit the collision?

Solve this problem just like (a), except for one thing: if the balls are stuck together, then there is only ONE final velocity for BOTH balls.

*** The final two problems are EXTRA CREDIT ***

c) If the two balls were ideally springy, so that they collided ***perfectly elastically***, with what instantaneous velocity would EACH ball exit the collision.
Hint 1: It will not necessarily be the same velocity for each ball.
Hint 2: A perfectly elastic collision conserves ***kinetic energy*** AND ***linear momentum***. So, for the entire 2-ball system, both the total value of ***mv*** and ***(1/2) mv***^{2} will remain the same before and after the collision.

d) Assume that all other originally given magnitudes are as stated above. Assume, however, that the pool ball was originally heading at 30 degrees North-East while the tennis ball was originally heading 30 degrees North-West. The collision is somewhat inelastic (neither perfectly elastic nor "sticky"). If the tennis ball exits the collision with an instantaneous velocity of 1 m/s due North, what is the exit velocity of the pool ball?
Hint 1: Draw a diagram of the situation.
Hint 2: Momentum is a vector.
Hint 3: So you can break it into components, just like any other vector.
Hint 4: Set up separate momentum-conservation equations for each axis.