Lab 6:
MidSemester
LAB PRACTiCUM

From Force to Work

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I. THE TRANSITION

FROM (THE STUDY OF) FORCE TO (THE STUDY OF) ENERGY

A. *Background*

The following exercise is provided to help us make a transition into a way of thinking that physicists started to consider (in the late 18th century) *after* they had accumulated some familiarity with both the predictive power and the limitations of *Newton's Laws of Motion* (published in the late 17th century). We will continue to consider and apply Newton's Laws in lecture – just as physics in general continues to emphasize and use Newton's Laws to this day. We will soon, however, begin to add the concepts of *conservation* and *energy* to our understanding of the universe. The transition starts in the lab – with today's investigation.

We will take a look at the simple pendulum. We will not assume any prior understanding of pendulums nor of concepts such as *energy*. We will simply assume a growing familiarity with forces (as well as general lab techniques). We will apply that familiarity to the simple pendulum until we start noticing some seemingly simple questions that seem difficult to answer with the techniques we have.

In order to ease this transition (from the study of *force and acceleration* to the study of *work and energy*), we will pause to review:

B. *Procedure*

Below, you will find a set of twenty statements – each of which needs to be assessed as either **true** or **false** (in the context of this class). Each lab group will be given approximately thirty minutes to discuss and achieve consensus on this entire T/F questionnaire (known as *The Excluded Middle*). Each group, that is, will discuss until it settles on one shared response to the entire set of twenty statements.

NOTE WELL:

The T/F questionnaire will be ***submitted*** by each group with that group's ***lab report***.

The T/F questionnaire, however, is often ***graded separately*** and treated as its own assignment with its own independent grade consequences. Your lab instructor will tell you precisely what to expect in terms of the grading context for this review/transition exercise.

The minimum expectation is a one-letter answer next to each statement. If there is any possibility or discussion of "extra credit", then the more careful and clear explanation (verbal, mathematical, pictorial) is provided for each T/F response, the more credit to be earned.

C. ***The Excluded Middle***

Put a "T" in the box next to each claim that appears logically ***consistent*** with Galileo's Principle of Relativity.
Put an "F" in the box next to each claim that appears logically ***in****consistent* with GPR.

1. The instantaneous speed of an object is its distance traveled divided by the time elapsed.	
	XXXXXX
2. When an object experiences constant acceleration, its average velocity is the total displacement divided by the total amount of time elapsed.	
	XXXXXX
3. The acceleration of an object can be (meaningfully) expressed by a negative number.	
	XXXXXX
4. If an object accelerates from rest at a constant rate of 20 feet/s^2, then it covers 90 feet in the first ***3*** seconds of travel.	
	XXXXXX
5. If an object's position/time graph is a straight but not horizontal line, then that object's acceleration/time graph will be a horizontal line.	
	XXXXXX
6. To compute the velocity of one object is to acknowledge the existence of a second object.	
	XXXXXX
7. Velocity is the magnitude of the speed vector.	
	XXXXXX
8. My velocity relative to you is the sum of my velocity relative to Earth and Earth's velocity relative to you.	
	XXXXXX
9. Projectile motion is an example of free-fall motion.	
	XXXXXX
10. The instantaneous speed of a projectile at its peak height is 0.	
	XXXXXX
11. A bullet shot from a horizontal gun and a bullet dropped from the exact same height will land at the same time.	
	XXXXXX
12. For a projectile fired with initial speed v_0 at angle θ (measured up from horizontal), the average vertical velocity is $v_0 \sin \theta$.	
	XXXXXX
13. At the peak height of a projectile's journey, the instantaneous acceleration is 0.	
	XXXXXX
14. According to Galileo's principle of relativity, Newton's 3 laws of motion should be applied in an accelerated reference frame.	
	XXXXXX

15. Net force can be represented by a properly drawn vector.	
	XXXXXX
16. The Earth pulls at a book sitting on a table with a force equal in magnitude and opposite in direction to the normal force exerted on the book by the table.	
	XXXXXX
17. Statement (16), above, is an example of Newton's 3rd law of motion.	
	XXXXXX
18. The magnitude of the static friction force is always directly proportional to the magnitude of the relevant normal force.	
	XXXXXX
19. Friction on an object always points opposite the direction of that object's motion.	
	XXXXXX
20. If there is a friction force between two objects, there must also be a normal force, but there can be a normal force without a friction force.	

II. THE INVESTIGATION…

OF A SIMPLE PENDULUM

A. *Ultimate Goals:*

1. To grow accustomed to a fundamental physics phenomenon: the swing of a simple, planar pendulum.
2. To determine the variable(s) on which the period of the pendulum depends.
3. To determine the precise nature (relationship, function) of the pendulum period's dependence.

B. *Suggested or Potential Research Questions:*

1. What factor or factors determine how much *time* is takes a simple pendulum to make one full cycle: one swing back and forth?

   Another way to ask the above question: On what *independent variable* or variables does the *period* (time for one full cycle) of a simple pendulum depend?

2. Why? (That is, how can an analysis of forces and distances explain the initially surprising result of RQ 1?)

3. In what specific way does the pendulum full-cycle time (*period*) depend on whatever factors(s) determine it?

   Another way to ask the above question: What precise mathematical function best describes the relationship between the *period* of a simple pendulum to its most observably determinative *independent variable*?

C. *Necessary Equipment*

1. Ringstand
2. String
3. Various Masses
4. Mass Hook (for attaching mass to swinging string)
5. Stopwatch
6. Meter Stick
7. Protractor

D. *What we know*

1. Kinematics
2. Newton's Laws
3. Objects in free-fall accelerate downward at a constant rate of 9.8 m/s^2

E. *What we don't know*

1. Anything about pendulums besides what Newton & Kinematics can tell us
2. Anything about energy, momentum, etc., including what those words even mean

F. ***Methods: Data Collection***

FIRST APPROACH: Forces & Acceleration

1) Set up a simple pendulum by hooking a mass to a string and dangling the string from a ring stand or similar upright apparatus.

2) Practice letting the pendulum undergo small, regular, smooth swings.

3) Once you have gotten used to the pendulum, hold the pendulum at a small angle from the fully vertical (relaxed) orientation. Carefully measure this angle. This angle should be ***smaller than 20 degrees.***

4) Record the angle here:

   Theta (1) = ____________________________________ degrees.

5) Carefully consider what you observe while the pendulum swings. Consider the instant immediately AFTER the mass is released.

6) Draw a SYSTEM SCHEMA for the pendulum mass at this instant in time. The mass at the end of the string (often known as the *bob*) is your "object of focus".

7) In English, write out every Newton's 3^{rd} Law PAIR of ("action-reaction") forces implied by your SYSTEM SCHEMA. Each "pair" should consist of two sentences. Each sentence will necessarily refer to two real, identifiable, objects. Each sentence will necessarily use either the word "pull" or the word "push". Each sentence will necessarily refer to some kind of direction – even if exotic or freshly motivated by the new situation presented by a pendulum. Examples such as "up", "down" "up/left", "along a radius in toward a center", "along a radius out toward a circumference", etc.

8) Draw a PURE FREE-BODY DIAGRAM of the pendulum mass at this instant in time (immediately after the mass is released). The mass at the end of the string (often known as the *bob*) is your "object of focus".

9) Now carefully consider and choose a coordinate system that you believe to be convenient or appropriate for this type of motion. How is your "x-axis" aligned? How is your "y-axis" aligned?

   HINT 1: In which direction do you believe the pendulum is ***accelerating*** at this moment?

   HINT 2: The direction of acceleration for an object traveling in a curve can be challenging to identify. Feel free to use the web and other resources to help you consider this question.

10) Once you have chosen a suitable coordinate system, break up any diagonal forces from your PURE F-B-D and thereby create a COMPONENT F-B-D for your pendulum bob.

11) Now that we have a component F-B-D, we have a great deal of information about the ***forces*** acting on a pendulum ***at any given instant*** during its swing.

We also still know and believe Newton's 2nd Law. We therefore know and/or can figure out the pendulum's ***acceleration*** at any instant. Find that acceleration.

12) We also still know and believe in relationships such as

$$x = 1/2at^2 + v_ot + x_0$$

And we have acceleration. And, oh hey, we have initial velocity too. What are we trying to find again? Oh, right. Time. So, what are we missing?

What displacement are we interested in? Well, we're trying to find the time required for one full ***period*** of the pendulum, in other words the time to complete one full swing back and forth.

Consider the bob's path as it swings. It swings from one side to the other. Assume that the angle it reaches on the far side is approximately equal to the angle it starts from. This path is not a straight line. But it is a fraction of a very familiar shape. And the angles can tell you exactly what fraction.

Come up with an expression for the distance traveled in one complete period.

13) So it would appear that we now have all the information necessary to calculate or predict the following:

If released at rest from a small angle away from the vertical, how much time will it take for the pendulum to make a full swing to the same angle on the other side of the vertical?

14) Consider the very fair, seemingly simple and commonly asked question in (13), above. It would ***appear*** that we have access to all the information needed to predict ***time*** for a given ***displacement*** (a fundamental physics question). Indeed, for any given instant in time, we do. Using Newton's Laws in order to predict time for a given pendulum displacement (or the other way around) is, however, far more difficult than it sounds. There is something about the pendulum – despite its simplicity and elegance of motion – that makes predicting times very challenging. Why so challenging? What is the obstacle to making straightforward use of Newton's Laws and kinematic equations?

As clearly and precisely as possible, explain how/why Newton's Laws and the kinematic equations can be true yet not particularly helpful in making space/time predictions for a pendulum. What is true about a pendulum that has not been true of any situation we have studied to this point?

Once you confidently see how Newton's Laws can be true for all mechanical situations, yet inconvenient for some, proceed to investigate time/space in a different way:

SECOND APPROACH: Cycles, Period, & Symmetry in Time

15) Release the pendulum from the above angle, theta (1), and measure the amount of time the pendulum takes to undergo 20 full (BACK-and-FORTH) swings.

16) Divide your answer by 20 to conclude the ***period, T,*** of your pendulum in seconds.

17) Record the period here: T (1) = ____ seconds.

18) In principle, you now have exactly one data point (one "ordered pair"). Your datum consists of one value for the dependent variable of period and one corresponding value for the independent variable of initial angle. You have no particular reason to believe that initial angle determines period at all – let alone in any specifically recognizable way. You are trying out this independent variable in order to see if there is any relationship at all. The only way you are going to find out is by trying out some other values – some other data points – for these variables. In a few minutes, you will do just that.

    BUT: Before you start building a possible pattern or relationship from a number of data points, it would be nice to reduce any possible error (yes *error*, not uncertainty) in your measurements. So, before moving on, do what we almost always do and what should now becoming an understood part of your approach to data collection:

    Repeat steps (15) through (17), above, at least two or three more times for the ***same angle*** you chose in step (4). That is, subject the *same measurement* to at *least three trials*. Take the results of these trials (all performed for the same value of independent variable) and average them. NOW, you truly have ONE comparatively reliable DATA POINT.

19) Repeat all the above procedure for two new angles (***Theta (2)*** and ***Theta (3)***). Make certain that these two angles are ALSO LESS THAN 20 DEGREES.

    You should have made approximately ***NINE*** separate measurements at this point—in order to find three data points. Each of the three data points consists of one value for period (dependent variable) discovered to arise when one value of initial angle (independent variable) has been chosen by you, the researcher. Each of the three variable values were tested three in three identical trials – simply to minimize ***error***.

20) ******* Set up a data chart correlating ***T*** with ***Theta***. It should have two columns and three rows of data. The units of measurement should be explicit somewhere.

21) Neatly, by hand or with computer software (*Excel*, *Logger Pro*, etc.), produce a simple but CLEAR graph of ***T*** (period of pendulum, measured in seconds) as a function of ***theta*** (initial angle from vertical). Both axes must be clearly labeled with units.

    The graph should consist of at least 3 data points and then an approximate "BEST-FIT" trend line capturing the trend of the data.

    DO NOT PLAY "connect-the-dots". The trend line should be a smooth (straight or curved) line.

All the above steps were designed to help you determine what relationship (if any) exists between pendulum ***period*** and ***initial*** pendulum ***angle***. NOW:

22) Create your own steps (modeled after the above) in order to gather data, create graphs, and determine the following:

a) Period as a function of Mass.
b) Period as a function of String Length.

For each of the above, you are repeating your method for data collection but applying it freshly to a new possible candidate for independent variable, first mass, then string length. Recall the scientific method. The idea is to hold *constant* as much of your universe as possible and just change *one quantity* at a time (tweak *one* deliberately chosen *independent variable*) so that you can study how nature responds through one specific dependent variable. One cause, one effect. One graph: consisting of two axes.

THEREFORE: ***When varying mass, for example, take all your different measurements at one and only one value for initial angle*** (any small angle you have a comparatively easy time measuring). If you start varying angle again, you're then looking at an interaction between two variables, which is (a) a much more complicated task and (b) not what we're trying to do here. Only vary one thing at a time.

By the time you are done with this step, you will have three different potential relationships expressed in three different graphs.

23) Given the basic shapes (trends) of your graphs, which ONE of the following candidates do you believe most observably determines the period of a simple pendulum?

Initial angle from *equilibrium* (the relaxed vertical orientation)? *Mass* of *bob* (the comparatively heavy object hanging from the bottom of the string)? *Length* of string?

Why?!

24) Go back to all of that beautiful Newton's Laws and kinematics work that you did in steps 5-14. Did you think that work was wasted? It wasn't.

Look at the distance and acceleration that you found in steps 11 and 12. Think about the relationship between speed, time and distance.

In 3-5 sentences of English, explain why the variable you selected in step 23 DOES have a significant effect on period and the other two do NOT. This will take some thinking.

25) So, you have your chosen dependent variable, and by looking at the forces on the system and the path traveled, you can see WHY this is the variable that has the greatest effect on period. But HOW does it effect period? What is the MATHEMATICAL relationship between the two?

In other words, we want to ***answer research question #2: we want to determine – as precisely as possible – a function to predict the period of a pendulum***.

26) In order to find this relation, we will need a few more data points. For whichever variable you picked in step 23, you currently have three data points. ***Pick at least 2 more values for this variable, perform three trials with each of the new values, and add the resulting two data points to your graph for the variable you selected***.

Make sure the values you are using for this variable include a ***large range***. Make sure that some are ***very small*** and others ***reasonably large***. The biggest value you test should be ***at least five times*** bigger than the smallest value you test.

We're now going to consider one more point for our graph. Think about what happens to period as length goes to zero. Does it seem like period will also approach zero? Does your data suggest that? If so, you might want to add the point 0,0 to your graph, even though that's not in your experimental trials (since you can't test a length of zero).

27) Now to find the mathematical relationship between the independent and dependent variable…

If this graph happens to be a straight line (constant slope), then we are quite fortunate; it will be comparatively straightforward to use – for prediction, retrodiction or, if you like, extrapolation.

But what if your graph is *not* linear?! (Hint: It's not.) Does that mean you have made a mistake somewhere?! (Hint: No. You have not. Unless you have.) Do we actually find non-linear relationships in nature?! (Hint: Yes. Way more frequently than we are lucky enough to stumble on linear relationships. We like linear because it's manageable, not because it's common.)

The first thing to do, if we are dealing with a non-linear graph is to figure out what TYPE of relation/function it represents. For example: does it look, generally speaking, like a sine curve? Like a parabola? Like an exponential growth function?

If you're not sure, open Desmos (or some other excellent graphing software, but what could be as good as Desmos?) try graphing some common to see what they look like.

28) Once you have a reasonable guess as to what general TYPE of function you are dealing with, the next step is to produce a curve of best fit using this function.

Using any software you like, try to find a best-fit curve for your data points, using the function you chose. A good tool for this is Desmos.

29) You should now have a mathematical function that relates period to your chosen variable. So you might be thinking, "I'm done, right?" Well, you're close, but no, not done.

See, we're doing physics, not pure math. We don't just want numbers. We want to relate them to physical properties. We want to understand what those numbers are made of.

Read on…

30) Your function will look something like

T = something × (some function on ***z***)

Where T is the period of the pendulum and z is the independent variable that you believe period most depends upon. For example, it might be

$$T = az^2$$

where ***a*** is some number with lots of decimal places. (It does ***not*** represent acceleration)

Now, your actual function won't actually have an z^2. It will have some other function on ***z***. But there will be a coefficient. There will be an ***a***. And if you think about it, this ***a*** has ***units***.

How do we know it has units? Well, ***T*** is in seconds, right? And ***z*** is in some other unit, not seconds. But somehow they end up equaling each other. So the ***a*** must have whatever units are necessary in order to turn the ***z*** units into the T units.

For example, say you found that

$$T = am^3$$

(which of course you didn't.) You know that T is in seconds and m^3 is in kg^3. So ***a*** must be in s/kg^3. That way am^3 will come out in seconds. See?

Find the correct units for the coefficient ***a*** in your function.

31) Now notice something about this coefficient ***a***: it is the constant part of your function. It's the one part that does NOT depend on angle or on length or on mass. It stays the same, no matter how you vary those variables. And it has units. Weird units. It must have something to do with the physical world.

Look back once again at all that work you did with Newton's Laws and kinematics at the beginning of the lab. Look at the equations you found for acceleration and distance for one period.

What constant of the physical world do you see in those equations that would probably have an effect on period?

Now ask yourself: given how it appears in the equation for acceleration or distance, would you expect it to have a direct correlation with period or an inverse correlation with period? In other words, if this constant of the physical world were to increase, would period tend to get longer or shorter?

The physical constant that you found also has UNITS. What are its units.

Now think hard. Think about the units of this physical constant. Think about whether it's directly or inversely related to period (and when you think inverse think 1/x). Think about the units you found for your coefficient in step 30. Think about what kind of math you have to do to that physical constant to get it in the same units as your coefficient.

32) Once you've figured out what kind of math has to be done to the physical constant to account for the strange units of your coefficient, you want to rewrite your function with this new parameter. Say your function was

$$T = \boldsymbol{a}z^3$$

and your physical parameter was M_e, the mass of the earth, (which it should ***not*** be) and you decided that, to get the right units, M_e has to be squared. Then you're going to rewrite your function as

$$T = \boldsymbol{b}M_e^2z^3$$

where ***b*** is some number. But this function should still fit your data. In other words, it should be equivalent to your old function. In other words $\boldsymbol{a}z^3$ must be equal to $\boldsymbol{b}M_e^2z^3$. You can literally set them equal and solve for ***b***. (Except of course that you will not have M_e^2 as your physical parameter. You'll have something that makes more sense.)

33) So, if you're really paying attention you might be thinking, wait a minute, we had this coefficient a. We did all this work to find out what physical parameter was inside it. And now we've got a new coefficient b. Now we have to figure out what physical parameter's in b? And then there'll be another coefficient c? And on and on forever?

But wait! There is an important difference between ***a*** and ***b***: ***a*** had units, ***b*** does not. ***b*** is a dimensionless number.

So there's no physical constant hidden inside of ***b***. But there might be some ***mathematical*** constant.

One last time, folks! We're almost done!

Go back to the acceleration and distance you found using Newton's Laws and kinematics. Look for some mathematical constant that looks like it will have something to do with the period. You might even want to think about ***two times this mathematical constant***, because that might have something to do with circumferences of circles and things of that sort.

See if two times that constant looks anything like your coefficient ***b***.

34) Given what you found above, write down your ultimate function: Write down the function that describes the relationship between the period of a pendulum and the length of the pendulum string. This relationship is your ultimate, conclusive, FINDING.

Your ultimate finding will be of the form: $\boldsymbol{T} = f(\boldsymbol{z})$

-- Where ***T*** is the period of the pendulum, and ***z*** is whatever independent variable you found to most evidently determine the period, and where your ***function***, ***f***, includes a combination of numerical and natural constants.