

The Universal Law of the Excluded Middle
or
Weapons of Mass Instruction
Cary Academy Debate: 8/12/5

Claim Types

SYNTHETIC	ANALYTIC
A POSTERIORI	A PRIORI

ANALYTIC: An analytic claim is a claim the truth or falsity of which is verifiable by means only of language (definitions).

SYNTHETIC: A synthetic claim is a claim that is **NOT** ANALYTIC.

A POSTERIORI: An a posteriori claim is a claim the truth or falsity of which is verifiable by means only of empirical (sensory) evidence.

A PRIORI: An a priori claim is a claim that is **NOT** A POSTERIORI.

Refutation Types

NON: A NON refutation is a refutation that asserts the strictly ANALYTIC opposite of the claim to be refuted. For example, if the given claim is “This chair is red,” then the NON refutation is “This chair is **not** red.” More rigorously, the NON refutation would be “It is **not** the case that this chair is red.” For any given claim, there is one and only one possible NON refutation.

POP: A pop refutation is a refutation that asserts some sort of SYNTHETIC opposite of the claim to be refuted. For example, if the given claim is “This chair is red,” then on example of a POP refutation might be “This chair is green.” For any given claim, there may be a wide array of possible POP refutations.

TRUTH TABLES

<u>P</u>	<u>Q</u>	<u>~P</u>	<u>~Q</u>	<u>P&Q</u>
T	T	F	F	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	F
<u>~(P&Q)</u>	<u>PvQ</u>	<u>~(PvQ)</u>	<u>P->Q</u>	<u>Q->P</u>
F	T	F	T	T
T	T	F	T	T
T	F	T	F	F
F	F	T	T	F
<u>~P->~Q</u>	<u>~Q->~P</u>	<u>P<->Q</u>	<u>P&~P</u>	<u>Pv~P</u>
T	T	T	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

PROOFS

1. $P \rightarrow Q, P \vdash Q$ [\rightarrow : Modus Ponens]

1.	$P \rightarrow Q$	A	1
2.	P	A	2
3.	Q	\rightarrow o 1,2	1,2
			QED

* *NOTE: This is* *NEITHER* $P \rightarrow Q, Q \vdash P$ $\vdash P$
 NOR $P \rightarrow Q, \sim P \vdash \sim Q$ $\vdash \sim Q$
 NOR $P \rightarrow Q \vdash Q$ $\vdash Q$. See Truth Table for \rightarrow .

2. $P \& Q \vdash P$ [$\&$]

1.	$P \& Q$	A	1
2.	P	$\&$ o 1	1
			QED

3. $P, Q \vdash P \& Q$ [$\&$ i]

1.	P	A	1
2.	Q	A	2
3.	$P \& Q$	$\&$ i 1,2	1,2
			QED

4. $P \rightarrow Q, P \& R \vdash Q$ [\rightarrow o, $\&$ o]

1.	$P \rightarrow Q$	A	1
2.	$P \& R$	A	2
3.	P	$\&$ o 2	2
4.	Q	\rightarrow o 1,3	1,2
			QED

5. $P \vdash P \vee Q$ [\vee i]

1.	P	A	1
2.	$P \vee Q$	\vee i 1	1
			QED

6. $P \rightarrow Q, P \& R \mid - Q$

[$\sim o$]

1.	$P \rightarrow Q$	A	1
2.	$P \& R$	A	2
3.	$\sim Q$	PA	3
4.	P	PPA	4
5.	Q	$\rightarrow o$ 1,4	1,4
6.	$Q \& \sim Q$	$\& i$ 3,5	1,3,4
7.	$\sim P$	$\sim i$ 4-6	1,3
8.	P	$\& o$ 2	2
9.	$P \& \sim P$	$\& i$ 7,8	1,2,3
10.	Q	$\sim o$ 3-9	1,2
			QED

7. $A \vee \sim A$

[$\sim o$: ULEM form #2]

1.	$\sim(A \vee \sim A)$	PA	1
2.	A	PPA	2
3.	$A \vee \sim A$	$\vee i$ 2	2
4.	$(A \vee \sim A) \& \sim(A \vee \sim A)$	$\& i$ 1,3	1,2
5.	$\sim A$	$\sim i$ 2-4	1
6.	$\sim A$	PPA	6
7.	$A \vee \sim A$	$\vee i$ 6	6
8.	$(A \vee \sim A) \& \sim(A \vee \sim A)$	$\& i$ 1,7	1,6
9.	A	$\sim o$ 6-8	1
10.	$A \& \sim A$	$\& i$ 5,9	1
11.	$A \vee \sim A$	$\sim o$ 1-10	---
			QED

8. $P \rightarrow R, Q \rightarrow R, P \vee Q \mid - R$

[$\vee o$]

1.	$P \rightarrow R$	A	1
2.	$Q \rightarrow R$	A	2
3.	$P \vee Q$	A	3
4.	R	$\vee o$ 1,2,3	1,2,3
			QED

9. $A \rightarrow P, \sim A \rightarrow P \mid - P$

[vo: ULEM form #2]

1.	$A \rightarrow P$	A	1
2.	$\sim A \rightarrow P$	A	2
3.	$\sim(A \vee \sim A)$	PA	3
4.	A	PPA	4
5.	$A \vee \sim A$	vi 4	4
6.	$(A \vee \sim A) \& \sim(A \vee \sim A)$	&i 3,5	3,4
7.	$\sim A$	$\sim i$ 4-6	3
8.	$\sim A$	PPA	8
9.	$A \vee \sim A$	vi 8	8
10.	$(A \vee \sim A) \& \sim(A \vee \sim A)$	&i 3,9	3,8
11.	A	$\sim o$ 8-10	3
12.	$A \& \sim A$	&i 7,11	3
13.	$A \vee \sim A$	$\sim o$ 3-12	---
14.	P	vo 1,2,13	1,2
			QED

10. $P \rightarrow (P \vee Q)$

[$\rightarrow i$]

1.	P	PA	1
2.	$P \vee Q$	vi 1	1
3.	$P \rightarrow (P \vee Q)$	$\rightarrow i$ 1-2	---
			QED

11. $P \rightarrow Q, Q \rightarrow P \mid - P \leftrightarrow Q$

[$\leftrightarrow i$]

1.	$P \rightarrow Q$	A	1
2.	$Q \rightarrow P$	A	2
3.	$(P \rightarrow Q) \& (Q \rightarrow P)$	&i 1,2	1,2
4.	$P \leftrightarrow Q$	$\leftrightarrow i$ 3	1,2
			QED

12. $P \leftrightarrow Q \mid - P \rightarrow Q$

[$\leftrightarrow o$]

1.	$P \leftrightarrow Q$	A	1
2.	$P \rightarrow Q$	$\leftrightarrow o$ 1	1
			QED

13. $P \rightarrow Q \mid \sim PvQ$

[$\rightarrow \mid \sim v$]

1.	$P \rightarrow Q$	A	1
2.	$\sim(\sim PvQ)$	PA	2
3.	P	PPA	3
4.	Q	$\rightarrow o$ 1,3	1,3
5.	$\sim PvQ$	vi 4	1,3
6.	$(\sim PvQ) \& \sim(\sim PvQ)$	$\& i$ 2,5	1,2,3
7.	$\sim P$	$\sim i$ 3-6	1,2
8.	$\sim P$	PPA	8
9.	$\sim PvQ$	vi 8	8
10.	$(\sim PvQ) \& \sim(\sim PvQ)$	$\& i$ 2,9	2,8
11.	P	$\sim o$ 8-10	2
12.	$P \& \sim P$	$\& i$ 7,11	1,2
13.	$(\sim PvQ)$	$\sim o$ 2-12	1
			QED

14. $(\sim PvQ) \rightarrow (P \rightarrow Q)$

[$v \mid \rightarrow$]

1.	$\sim PvQ$	PA	1
2.	P	PPA	2
3.	$\sim P$	PPPA	3
4.	$\sim Q$	PPPPA	4
5.	$P \& \sim P$	$\& i$ 2,3	2,3
6.	Q	$\sim o$ 4-5	2,3
7.	$\sim P \rightarrow Q$	$\rightarrow i$ 3-6	2
8.	Q	PPPA	8
9.	$Q \& P$	$\& i$ 2,8	2,8
10.	Q	$\& o$ 9	2,8
11.	$Q \rightarrow Q$	$\rightarrow i$ 8-10	2
12.	Q	vo 1,7,11	1,2
13.	$P \rightarrow Q$	$\rightarrow i$ 2-12	1
14.	$(\sim PvQ) \rightarrow (P \rightarrow Q)$	$\rightarrow i$ 1-13	---
			QED

15. $\sim(P \& Q) \vdash \sim P \vee \sim Q$

[$\&$ \vdash v: DeMorgan's Law form #1]

1.	$\sim(P \& Q)$	A	1
2.	$\sim(\sim P \vee \sim Q)$	PA	2
3.	$\sim P$	PPA	3
4.	$\sim P \vee \sim Q$	vi 3	3
5.	$(\sim P \vee \sim Q) \& \sim(\sim P \vee \sim Q)$	$\&i$ 2,4	2,3
6.	P	$\sim o$ 3-5	2
7.	$\sim Q$	PPA	7
8.	$\sim P \vee \sim Q$	vi 7	7
9.	$(\sim P \vee \sim Q) \& \sim(\sim P \vee \sim Q)$	$\&i$ 2,8	2,7
10.	Q	$\sim o$ 7-9	2
11.	$P \& Q$	$\&i$ 6,10	2
12.	$(P \& Q) \& \sim(P \& Q)$	$\&i$ 1,11	1,2
13.	$\sim P \vee \sim Q$	$\sim o$ 2-12	1
			QED

16. $(\sim P \vee \sim Q) \rightarrow \sim(P \& Q)$

[\vee \vdash $\&$: Converse, DeMorgan's Law form #1]

1.	$\sim P \vee \sim Q$	PA	1
2.	$P \& Q$	PPA	2
3.	P	$\&o$ 2	2
4.	$\sim P$	PPPA	4
5.	Q	PPPPA	5
6.	$P \& \sim P$	$\&i$ 3,4	2,4
7.	$\sim Q$	$\sim i$ 5-6	2,4
8.	$\sim P \rightarrow \sim Q$	$\rightarrow i$ 4-7	2
9.	$\sim Q$	PPPA	9
10.	$\sim Q \& (\sim P \vee \sim Q)$	$\&i$ 1,9	1,9
11.	$\sim Q$	$\&o$ 10	1,9
12.	$\sim Q \rightarrow \sim Q$	$\rightarrow i$ 9-11	1
13.	$\sim Q$	$\vee o$ 1,8,12	1,2
14.	Q	$\&o$ 2	2
15.	$Q \& \sim Q$	$\&i$ 13,14	1,2
16.	$\sim(P \& Q)$	$\sim i$ 2-15	1
17.	$(\sim P \vee \sim Q) \rightarrow \sim(P \& Q)$	$\rightarrow i$ 1-16	---
			QED

17. $(P \rightarrow Q) \rightarrow \sim(P \& \sim Q)$

[\rightarrow | $\&$]

1.	$P \rightarrow Q$	PA	1
2.	$P \& \sim Q$	PPA	2
3.	P	$\&o$ 2	2
4.	$\sim Q$	$\&o$ 2	2
5.	Q	$\rightarrow o$ 1,3	1,2
6.	$Q \& \sim Q$	$\&i$ 4,5	1,2
7.	$\sim(P \& \sim Q)$	$\sim i$ 2-6	1
8.	$(P \rightarrow Q) \rightarrow \sim(P \& \sim Q)$	$\rightarrow i$ 1-7	---
			QED

18. $\sim(P \& \sim Q) \rightarrow (P \rightarrow Q)$

[$\&$ | \rightarrow]

1.	$\sim(P \& \sim Q)$	PA	1
2.	P	PPA	2
3.	$\sim Q$	PPPA	3
4.	$P \& \sim Q$	$\&i$ 2,3	2,3
5.	$(P \& \sim Q) \& \sim(P \& \sim Q)$	$\&i$ 1,4	1,2,3
6.	Q	$\sim o$ 3-5	1,2
7.	$P \rightarrow Q$	$\rightarrow i$ 2-6	1
8.	$[\sim(P \& \sim Q)] \rightarrow (P \rightarrow Q)$	$\rightarrow i$ 1-7	---
			QED

19. $(P \rightarrow Q) \rightarrow (\sim Q \rightarrow \sim P)$

[\rightarrow | \leftarrow : Contrapositive]

1.	$P \rightarrow Q$	PA	1
2.	$\sim Q$	PPA	2
3.	P	PPPA	3
4.	Q	$\rightarrow o$ 1,3	1,3
5.	$Q \& \sim Q$	$\&i$ 2,4	1,2,3
6.	$\sim P$	$\sim i$ 3-5	1,2
7.	$\sim Q \rightarrow \sim P$	$\rightarrow i$ 2-6	1
8.	$(P \rightarrow Q) \rightarrow (\sim Q \rightarrow \sim P)$	$\rightarrow i$ 1-7	---
			QED