A Thing on a Spring:
Transition Problem #1 of 2

PHYSICS 204:
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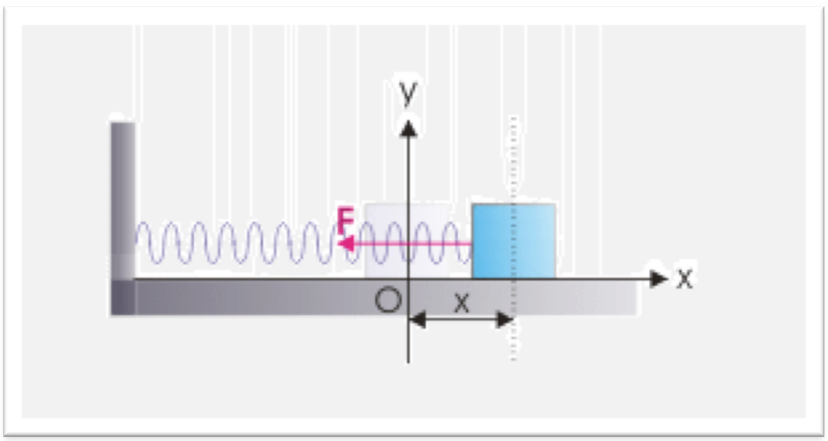
A mass is fixed to one end of a horizontal spring. The other end of the spring is fixed to a wall. The spring is "ideal": Its mass is insignificant when compared to that of the mass, it is coiled tightly and it is far longer than any distance the mass will ever move. The spring, that is, obeys Hooke's Law.



The mass is 300 grams (0.3 kg). The spring has a stiffness given by the constant K = 200 Newtons/meter. The mass is initially displaced 15 cm (0.15 m) from equilibrium. It is held at rest and then RELEASED. The mass therefore starts traveling in—toward the equilibrium position. The equilibrium position is designated x = 0 m.

1. What is the force exerted by the spring on the mass the instant it is released?

2. What is the acceleration of the mass the instant it is released?

3. What is the acceleration of the mass as it passes through the equilibrium position?

4. In meters/sec, at what instantaneous speed does the mass travel through the equilibrium position?

5. In meters, at what position does the mass achieve an instantaneous speed of 3 meters/sec?

6. In meters, how **far** will the mass compress the spring once the mass has instantaneously stopped and is about to reverse direction?

7. Assuming that the amount of friction is too small to make a measurable difference, how **many** indistinguishable cycles can this mass complete until it is incapable of moving?

8. As the mass travels from 15 cm toward the equilibrium, what happens to the magnitude of…
   a. The VELOCITY (increase, decrease or remain constant)?
   b. The ACCELERATION (increase, decrease or remain constant)?
   c. The DERIVATIVE of ACCELERATION (i.e. "JERK") (increase, decrease or remain constant)?

9. **** CHALLENGING BUT CENTRAL ****

In meters, WHERE will the mass be at precisely t = 1 sec
(1 second after it is released from rest)?

Hint A: Can you use $x = \frac{1}{2} at^2 + v_o t$? Why or why not?
Hint B: Strongly consider the option why not.
Hint C: Under what conditions CAN you use the equation given in Hint A? Under what conditions can you NOT use that function?

Hint D: It may now seem that you are stuck. There IS a solution, but it's TRICKY!

What we need to do is relate ***position*** to ***time***.
In other words, we need an equation that gives ***position*** as a function of ***time***.
Try the steps below:

Step 1: Go back to Hook's Law. This Law gives force as a function of position.
Step 2: Use NII to rewrite this as acceleration as a function of position.
Step 3: How are acceleration & position related (in calculus language)?
Step 4: Replace acceleration with the _____ of position.
Step 5: What you are looking at is called a 2nd order differential equation.
Step 6: If you can solve this differential equation, you will have the function you need.
Step 7: What ***type*** of equation would you expect to describe the motion of a pendulum with respect to time.
Step 8: Feel free to use the textbook, the internet or any other resources to help— but make sure that you can EXPLAIN the work you did… in other words, don't just copy, comprehend!