Mathematical Methods

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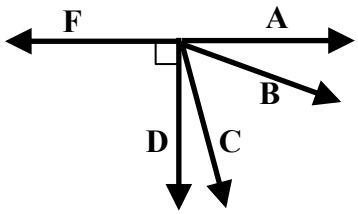
I. VECTOR MULTIPLICATION:

The problems below refer to the following vectors, all of which have equal magnitudes:



1. Put the following dot-products in order from greatest to smallest. Please give some explanation of your reasoning:

   a. $\vec{A}\cdot\vec{B}$ b. $\vec{A}\cdot\vec{C}$ c. $\vec{B}\cdot\vec{C}$ d. $\vec{A}\cdot\vec{D}$

2. Put the following cross-products in order from greatest to smallest magnitude. Please give some explanation of your reasoning:

   a. $\vec{A}\times\vec{B}$ b. $\vec{A}\times\vec{C}$ c. $\vec{A}\times\vec{D}$ d. $\vec{A}\times\vec{F}$

3. Assuming that all the vectors have magnitude 3, find the following (if the result is a vector, indicate the direction):

   a. $\vec{A}\cdot\vec{F}$ b. $\vec{A}\cdot\vec{D}$ c. $\vec{A}\times\vec{F}$ d. $\vec{A}\times\vec{D}$ e. $\vec{F}\times\vec{D}$

   f. $\vec{C}\cdot\vec{C}$ g. $\vec{C}\times\vec{C}$

4. Assuming that $\vec{A}$ and $\vec{B}$ are separated by an angle of $30°$, and $\vec{A}$ and $\vec{C}$ are separated by an angle of $75°$, find the following (if the result is a vector, indicate direction):

   a. $\vec{A}\cdot\vec{B}$ b. $\vec{A}\cdot\vec{C}$ c. $\vec{B}\cdot\vec{C}$

   d. $\vec{A}\times\vec{B}$ e. $\vec{B}\times\vec{A}$ f. $\vec{B}\times\vec{C}$

   b. $\vec{A}\cdot\hat{C}$ b. $\hat{D}\cdot\hat{C}$ c. $\hat{F}\times\vec{C}$

II. DERIVATIVES

A) Given $x = A\cos(\omega t)$, find:

i. $\frac{dx}{dt} =$

ii. $\frac{d^2x}{dt^2} =$

B) Is $x = A\cos(\omega t)$ a solution to $\frac{d^2x}{dt^2} = -\omega^2 x$? Why or why not?

C) Is $x = A\cos(5t)$ a solution to $\frac{d^2x}{dt^2} = -3x$? Why or why not?

D) Given $x = A\cos(5t)$, find:

i. $\frac{dx}{dt} =$

ii. $\frac{d^2x}{dt^2} =$

E) Given $x = A\cos(\sqrt{k/m}\cdot t)$, find:

iii. $\frac{dx}{dt} =$

iv. $\frac{d^2x}{dt^2} =$

F) Given $x = A\cos(\omega t + 3.5)$, find:

iii. $\frac{dx}{dt} =$

iv. $\frac{d^2x}{dt^2} =$

G) Is $x = A\cos(\omega t + 3.5)$ a solution to $\frac{d^2x}{dt^2} = -\omega^2 x$? Why or why not?

H) Given $x = e^{-\omega t}$, find:

i. $\frac{dx}{dt} =$

ii. $\frac{d^2x}{dt^2} =$

I) Is $x = e^{-\omega t}$ a solution to $\frac{d^2x}{dt^2} = -\omega^2 x$? Why or why not?

J) Given $x = e^{i\omega t}$ ($i \equiv \sqrt{-1}$), find:

i. $\frac{dx}{dt} =$

ii. $\frac{d^2x}{dt^2} =$

K) Is $x = e^{i\omega t}$ a solution to $\frac{d^2x}{dt^2} = -\omega^2 x$? Why or why not?

III. COSINE FUNCTIONS

A) Given $x = 4\cos(\pi t)$,

i. At t = 0, what will be the value of x?

ii. Find two values of t for which x = 0.

iii. Find two values of t for which x = 4.

iv. Find a value of t for which x = -4.

v. Find a value of t for which x = 2.

vi. What is the maximum possible value of x?

B) Given $x = 4\cos\left(2\pi t + \frac{\pi}{2}\right)$,

vii. When t = 0, what is the value of x?

viii. Find a value of t for which x = 4.

ix. Find a value of t for which x = 0.