HW1: A Thing on a Spring

P204 , SU16

1.

Energy is Conserved.

$$\left(\frac{1}{2}kx^2 + \frac{1}{2}mv^2\right)_H = \left(\frac{1}{2}kx^2 + \frac{1}{2}mv^2\right)_T$$

Let $_H \equiv$ *'Here'*, the position from which the mass is released.

Let $_T \equiv$ *'There'*, the 'Equilibrium' position about which we are being asked.

NOTE (for life in general):

An 'Equilibrium' position is, by definition,

a point in space at which

$\sum F = 0$

and therefore at which

$\boxed{a = 0.}$

Equilibrium is NOT necessarily a place at which $v = 0$.

It couldn't be.

Velocity never depends place, but rather on *Frame of Reference.*

$$Then \ldots$$

$$\left(\frac{1}{2}kx^2 + \cancelto{0}{\frac{1}{2}mv^2}\right)_H = \left(\cancelto{0}{\frac{1}{2}kx^2} + \frac{1}{2}mv^2\right)_T$$

$$\left(\cancel{\frac{1}{2}}kx^2\right)_H = \left(\cancel{\frac{1}{2}}mv^2\right)_T$$

$$m{v_T}^2 = k{x_H}^2$$

$$m{v_T}^2 = k{x_H}^2$$

$${v_T}^2 = \frac{k{x_H}^2}{m}$$

$$v_T = \sqrt{\frac{k{x_H}^2}{m}}$$

$$= \sqrt{\frac{(200)(.15)^2}{.3}}$$

$$= \sqrt{15}$$

$$\approx 3.87\, m/s$$

$$\boxed{v \approx 3.87\, m/s}$$

2.

Energy is Conserved.

$$(\frac{1}{2}kx^2 + \frac{1}{2}mv^2)_H = \left(\frac{1}{2}kx^2 + \frac{1}{2}mv^2\right)_T$$

Let $_H \equiv$ '*Here*', the position from which the mass is released.

(Equilbrium would work too — — any place about which we have full information.)

Let $_T \equiv$ '*There*', the $UNKNOWN$ position about which we are being asked.

Then ...

$$Then \ldots$$

$$\left(\frac{1}{2}kx^2 + \cancel{\frac{1}{2}mv^2}^{\,0}\right)_H = \left(\frac{1}{2}kx^2 + \frac{1}{2}mv^2\right)_T$$

$$\left(\cancel{\frac{1}{2}}kx^2\right)_H = \left(\cancel{\frac{1}{2}}kx^2 + \cancel{\frac{1}{2}}mv^2\right)_T$$

$$kx_T^2 = kx_H^2 - mv^2$$

$$x_T^2 = \frac{kx_H^2 - mv^2}{k}$$

$$x_T = \sqrt{\frac{kx_H^2 - mv^2}{k}}$$

$$= \sqrt{\frac{200(.15)^2 - .3(3)^2}{200}}$$

$$= \sqrt{\frac{4.5 - 2.7}{200}}$$

$$= \sqrt{\frac{1.8}{200}} = \sqrt{.009}$$

$$\approx .095\,m$$

$$\boxed{x \approx .095\,m}$$

DOES YOUR INTUITION SUGGEST SOMETHING LIKE

.15 METERS ON THE OTHER SIDE?

Fine — Now LET'S HANDLE OUR INTUITION WITH CARE AND DELICACY:

IT'S AMONG OUR MOST PRIZED POSSESSIONS...

(i.e.: CHECK.)

Energy is Conserved.

$$\left(\frac{1}{2}kx^2 + \frac{1}{2}mv^2\right)_H = \left(\frac{1}{2}kx^2 + \frac{1}{2}mv^2\right)_T$$

Let $_H \equiv$ '*Here*', the position from which the mass is released.

Let $_T \equiv$ '*There*', the 'Equilibrium' position about which we are being asked.

$$\left(\frac{1}{2}kx^2 + \cancelto{0}{\frac{1}{2}mv^2}\right)_H = \left(\frac{1}{2}kx^2 + \cancelto{0}{\frac{1}{2}mv^2}\right)_T$$

Yup: $x^2 = .0225$

The two mathematical roots are the two points in space at which $v = 0$:

The two 'turning points'.

GOOD THING, YOUR INTUITION!

NOTE (for life in general):

At any 'Turning Point' in an energy conserving system (like this one),

Kinetic Energy is 0

and Potential Energy is a maximum.

5.

a. Magnitude of Velocity (i.e.: 'Speed'):

INCREASES

b. Magnitude of Acceleration (i.e.: 'Acceleration Magnitude'):

DECREASES

c. Magnitude of Jerk (i.e.: 'Hipster Chicken Dish'):

INCREASES *

*Why? Good question.

Simplest answer:

Think about what must be the case whenever an object turns around.

If that doesn't help, see below (6).

6.

$$At\, t = 1, find\ x.$$

CANNOT use constant acceleration equation because acceleration is not constant (!)

What do we know?

$$\mathbf{F} = -\mathbf{kx}$$

$$\sum \mathbf{F} = \mathbf{ma}$$

Free − Body Diagram of mass on spring has but ONE (horizontal arrow)

So, here:

$$-\mathbf{kx} = \mathbf{ma}$$

$$ma = -kx$$

$$but$$

$$a \equiv \frac{d^2x}{dt^2}$$

$$So,$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\boxed{\frac{d^2x}{dt^2} = -(\frac{k}{m})x}$$

(k/m) is Just a CONSTANT; x and t are variables.

We could call that constant anything we want.

For nostalgia, let's call it: ω^2

We'll re − write the above

$$\boxed{\frac{d^2x}{dt^2} = -(\omega^2)x}$$

A solution is:

$$x = A\cos(wt)$$

for which $\omega = \sqrt{\frac{k}{m}}$

Here. A = .15 m

so $x = .15 \cos(\sqrt{\frac{200}{.3}})$

$$x = .15\cos(25.8 \text{ RADIANS})$$

$$\boxed{x \approx .117\,m}$$