MidTerm:

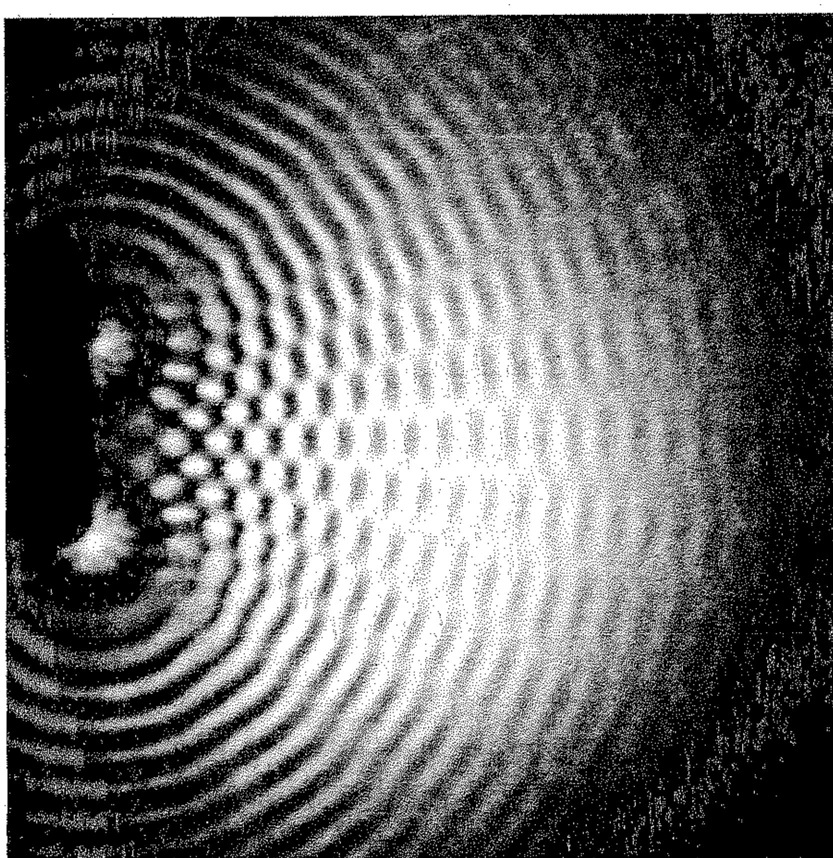
APPROVED BY YAVERBAUM

Propagation

PHYSICS 204, DANIEL A. MARTENS YAVERBAUM

JOHN JAY COLLEGE OF CRIMINAL JUSTICE, THE CUNY

WEDNESDAY, 4/2/14



Name: SOLUTIONS

Section: 10^{-19}

SCORE: ______________

THE FOLLOWING RELATIONS UNDERLY THE MATERIAL:

1) $\sum \vec{F} = m\vec{a}$.

2) $F = -Kx$.

3) $x = Acos(\omega t)$.

4) $\omega = 2\pi f$.

5) $f = \frac{1}{T}$.

6) $\theta \equiv \frac{x}{r}$.

7) $\lim_{\theta \to 0} \frac{sin\theta}{\theta} = 1$.

8) $v = \lambda f$.

9) $v = \sqrt{\frac{T}{\mu}}$.

10) $\frac{\partial^2 y}{\partial t^2} = (v^2)\frac{\partial^2 y}{\partial x^2}$.

11) $y = Acos(\omega t - kx)$.

12) $L = \frac{n\lambda}{2}$.

13) $\Delta l = \frac{n\lambda}{2}$.

14) $F = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{Q_1 q_2}{r^2}$.

15) $\vec{E} \equiv \frac{\sum \vec{F}}{q}$.

16) $\epsilon_0 \approx 8.85\ x\ 10^{-12}\ \frac{C^2}{Nm^2}$.

17) $m_e \approx 9.11\ x\ 10^{-31}$ kg.

18) $q_e \approx 1.60\ x\ 10^{-19}\ C$.

SCORE: ________________

<u>DIRECTIONS</u>

This test appears to be worth 125 pts.

You are only responsible, however, ONLY: for choosing and completing any 100 of the points.

All tests will be graded out of 100.
You may ignore 1/5 of this test and still conceivably achieve a perfect score.

You may choose to leave blank ANY 25 POINTS you wish.
You will not be penalized.

THERE IS ONE CATCH: If you earn a total of 0 points on ***any one full page***, then:

5 additional points will be deducted from your total raw score
(for each and every page on which you earn 0 points).

In other words:

Skip any 25 points that you chose – without penalty.

But do not skip nor be wrong on any <u>one entire page</u>.

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Finally, ***Note***:

If a question requires a numerical answer, you must provide one--even if you choose to skip questions directly before it. You cannot expect the grader to connect intellectual dots for you. You must make wise and strategic decisions.

SCORE: ______________

I. Simple Harmonic Oscillation (25 pts).

A. In the three sub-parts below, you are going to show that a particle subject only to the restoring force of an ideal spring oscillates cosinusoidally in time.

Assume that the particle has mass m,
the spring exerts a linear restoring force of strength K and that
the particle is initially held stationary at position A.

i. Further, Assume that Newton's Second Law of Motion and
Hooke's Law for ideal springs
apply to this mass/spring system.

Express Hooke's Law as a second-order differential equation and show that $x = A\cos\omega t$ is a general solution to this differential equation (5 pts)

$$F = -KX$$
$$\sum F = ma$$
$$-KX = ma$$
$$ma = -KX$$
$$a = -\left(\frac{K}{m}\right)X$$
$$\frac{d^2x}{dt^2} = -\left(\frac{K}{m}\right)X$$

$$X = A\cos(\omega t)$$
$$\frac{dx}{dt} = -A\omega\sin(\omega t)$$
$$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t)$$

$$+\cancel{A}\omega^2\cancel{\cos(\omega t)} = +\left(\frac{K}{m}\right)\cancel{A}\cancel{\cos(\omega t)}$$

So $X = A\cos\omega t$ IS A SOLUTION TO THIS DIF. EQ.

IFF $\omega = \frac{K}{m}$

SCORE: ______________

ii. As a function of K and m, derive the angular frequency of the particle, ω, such that the particle oscillates cosinusoidally in time. (Given your work in (i), above, this will require only a line or two. It might even be contained in your work above. Just copy the appropriate lines.) (2 pts).

$$\omega = \sqrt{\frac{K}{m}}$$

iii. From the particle's angular frequency, found in (i) above, determine the particle's period of oscillation, T, as a function of K and m (2 pts).

$$\omega \equiv 2\pi f \rightarrow f = \frac{\omega}{2\pi} \text{ and } T \equiv \frac{1}{f} \rightarrow T = \frac{1}{\omega}$$

$$\rightarrow T = \frac{2\pi}{\omega} \text{ ; so, } T = 2\pi\sqrt{\frac{m}{K}}$$

SCORE: ______________

ssume a particle of $m = 250$ gram particle is on an ideal spring of constant $K = 150$ N/m.

Assume also that the particle is *initially held at rest* at 7 cm from the spring's uilibrium position.

The mass is then released.

i. Find how much time it takes the particle to complete precisely ***three full cycles*** (3 pts).

$T = 2\pi\sqrt{m/K}$

$3T = 6\pi\sqrt{\frac{m}{K}}$

$= 6\pi\sqrt{\frac{.250}{150}} \approx 6\pi(.041\,sec)$

$\boxed{3T \approx .773\,sec}$

ii. In m/s^2, find the particle's acceleration at the instant it has completed 1/8 of its first cycle (4 pts).

$\omega = \sqrt{\frac{K}{m}}$

$= \sqrt{\frac{150}{.250}}$

$\omega \approx 24.5\ RAD/s$

$\approx 24.5\ s^{-1}$

$t = \frac{3T}{24} \approx .0322$

$X = A\cos(\omega t)$

$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t) = -.07(24.5)^2\cos[(24.5)(.0322)]$

$= -29.6\ m/s^2$

$\boxed{a \approx -29.6\ \frac{m}{s^2}}$

SCORE: ______________

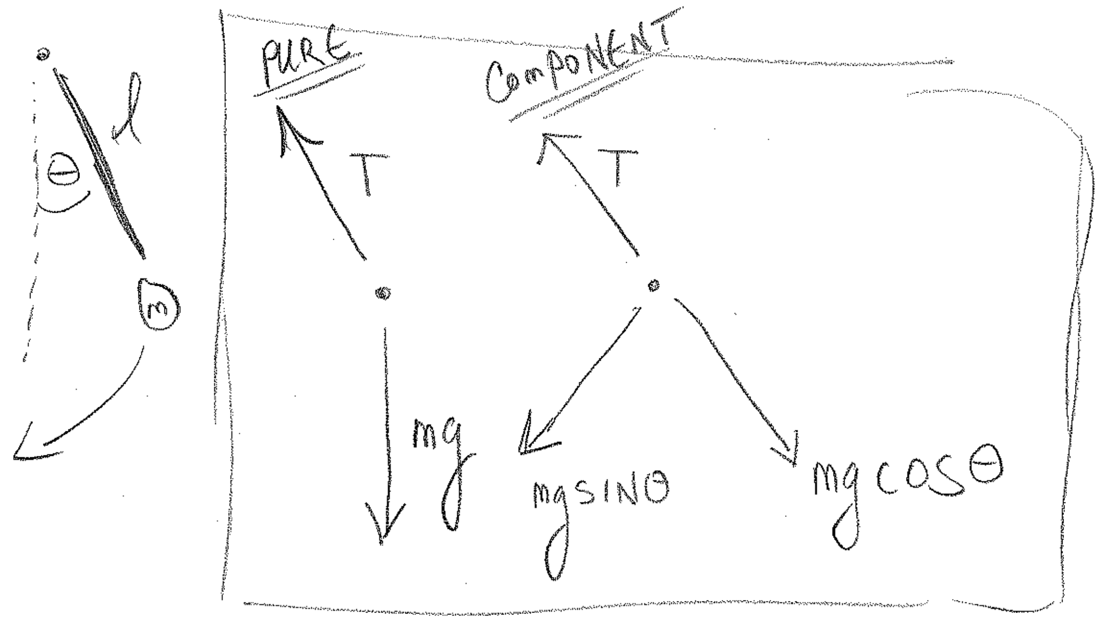
iii. Draw two free-body diagrams of a simple planar pendulum:

mass $\boldsymbol{m}$,

length $\boldsymbol{l}$,

displaced by some small angle, θ, from its equilibrium orientation.

One F-B-D should be "pure" and the other should be broken up into components that lie along some convenient set of coordinate axes (4 pts).



iv. Assume that the sine of an angle approaches the angle itself (measured in radians) for small angles. Applying Newton's 2[nd] Law and the definition of an angle (measured in radians) to your diagram, show that the small-angle oscillation of a pendulum should be approximately simple harmonic. (5 pts). $\theta \equiv \frac{X}{R}$

$$\sum \vec{F} = m\vec{a}$$

X ("RADIAL") AXIS

$$\sum F_R = m a_R$$

$$T - mg\cos\theta = m a_R$$

Y ("TANGENTIAL") AXIS

$$\sum F_T = m a_T$$

$$-\cancel{m}g\sin\theta = \cancel{m}a_T$$

$$a = -g\sin\theta$$

$$\frac{d^2x}{dt^2} = -g\sin\theta$$

SO $X \equiv \theta R$

$X = \theta l$

$$\frac{d^2(\theta l)}{dt^2} = -g\sin\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta$$

SCORE: ______

$$\frac{d^2\theta}{dt^2} \approx -\left(\frac{g}{l}\right)\theta \; : \text{FORM } \frac{d^2z}{dt^2} \approx -z \text{ SHO}$$

!

II. Electrostatic Oscillation (25 pts).

A large, solid, ***insulating*** sphere has negative charge **$-Q$** distributed uniformly throughout its volume. In other words, the 3-D charge density, q/V, is a constant. (You can call this constant ρ if you like.)

The radius of the charged sphere is R.

A small test charge, $+q$ is placed somewhere in the body at an arbitrary displacement from the center, r.

A) Derive an expression that describes how the electrostatic force exerted on $+q$ by $-Q$ varies as a function of r (7 pts).

SHOW ALL WORK:

IN WHATEVER WAY YOU FIND MOST COMFORTABLE, EXPLAIN ALL REASONING.

A CLEARLY LABELED DIAGRAM IS WORTH 2 out of the 7 pts.

$$\frac{q_{(enc)}}{V_{(enc)}} = \frac{Q}{V_{(TOT)}}$$

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 q_2}{r^2}$$

$$\frac{q_{(enc)}}{4/3\pi r^3} = \frac{-Q}{4/3\pi R^3}$$

$$q_{(enc)} = \frac{-Q r^3}{R^3}$$

$$\boxed{\text{so } F = \frac{-1}{4\pi\epsilon_0}\left(\frac{qQr}{R^3}\right)}$$

THE ELECTROSTATIC FORCES FROM ALL THE NEGATIVE POINT CHARGES outside $+q$ CANCEL ONE ANOTHER AND YIELD A NET ELECTROSTATIC FORCE of 0. ONLY THE NET EFFECT of ALL THE CHARGES WITHIN r MATTERS: TOGETHER, THEY ACT AS A POINT CHARGE LOCATED AT THE CENTER. MAGNITUDE of THIS POINT CHARGE?

$$\frac{q_{(enc)}}{V_{(enc)}} = \frac{Q}{V_{(TOTAL)}} \rightarrow$$

SCORE: ______________

B) Assume that $-Q = -55$ microC and $R = 20$ cm (very large ball of very large charge!).

If that small test charge, $+q$, is placed somewhere on the surface of the sphere and then allowed to fall all the way through an evacuated diameter:

How much time will it take for the test charge to reach the CENTER of the sphere (8 pts)?

$$F = -\frac{1}{4\pi\epsilon_0}\left(\frac{Qq}{R^3}\right)r \longrightarrow ma = -\left(\frac{1}{4\pi\epsilon_0}\cdot\frac{Qq}{R^3}\right)r$$

$$m\frac{d^2r}{dt^2} = -\frac{1}{4\pi\epsilon_0}\left(\frac{Qq}{R^3}\right)r \longrightarrow \frac{d^2r}{dt^2} = -\underbrace{\left(\frac{1}{4\pi\epsilon_0}\frac{Qq}{mR^3}\right)}_{\text{S.H.O.!!}}r$$

$$\omega^2 = \left(\frac{1}{4\pi\epsilon_0}\cdot\frac{Qq}{mR^3}\right) \rightarrow \omega = \left(\frac{1}{4\pi\epsilon_0}\cdot\frac{Qq}{mR^3}\right)^{\frac{1}{2}} \longrightarrow T = 2\pi\sqrt{\frac{mR^3\cdot 4\pi\epsilon_0}{Qq}}$$

$$\frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{mR^3\cdot 4\pi\epsilon_0}{Qq}} \qquad \frac{m}{q} = \frac{m_e}{e} = \frac{9.11\times10^{-31}\text{ kg}}{1.60\times10^{-19}\text{ C}} \approx 5.69\times10^{-12}\text{ kg/c}$$

or: $\left(K = \frac{1}{4\pi\epsilon_0}\frac{Qq}{R^3}\right)$

$\left(4\pi\epsilon_0 \approx 1.11\times10^{-10}\frac{C^2}{Nm^2}\right)$

$$\text{so } \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{(5.69\times10^{-12}\text{ kg/c})(.2\text{ m})^3}{(8.99\times10^{9}\frac{Nm^2}{C^2})(55\times10^{-6}C)}} \approx \boxed{4.77\times10^{-10}\text{ sec} \approx \frac{T}{4}}$$

SCORE: ______________

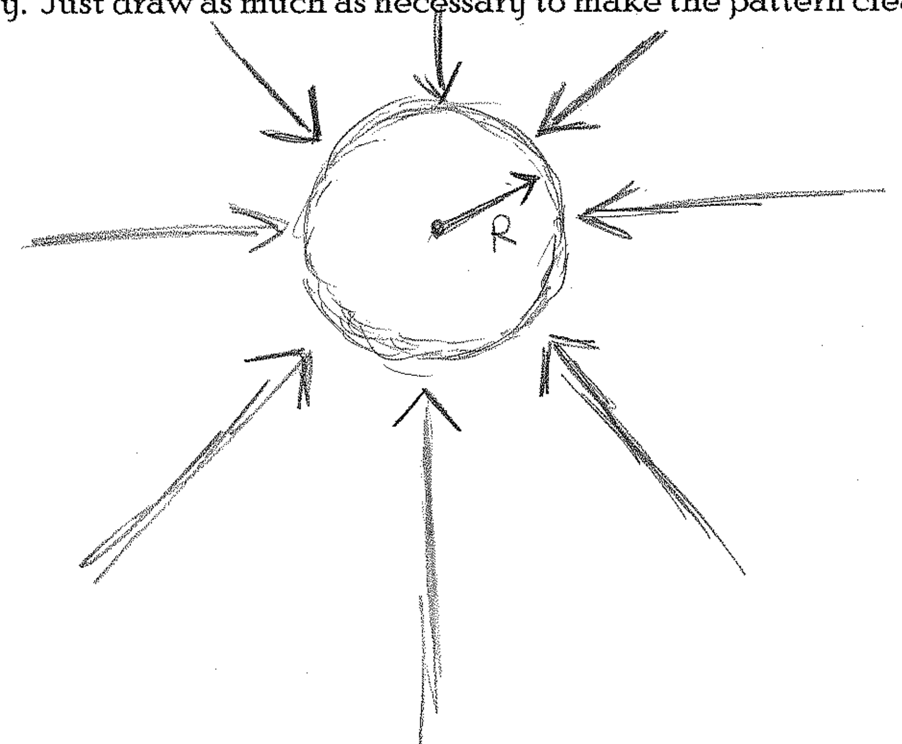
C) Compute the ***magnitude*** (in Newtons/Coulomb) and the ***direction*** (using some descriptive word such as "up", "down", "in", "out", "tangential", etc.) of the ***Electrostatic Field*** at a position 5 cm away from the center of the sphere (6 pts).

$$E = \frac{\Sigma F}{q} = -\frac{1}{4\pi\epsilon_0}\frac{Q}{R^3}r$$

$$= \frac{\left(8.99\times10^{9}\ \frac{N\,m^2}{C^2}\right)\left(55\times10^{-6}\ C\right)\left(.05\,m\right)}{(.20\,m)^3}$$

$E \approx 3.09\times10^{6}\ \frac{N}{C}$, IN TOWARD CENTER (ALONG radial line)

D) Assume the sphere described above is made of ***conducting*** material and is ***stable***. Draw a diagram of the field lines produced by this sphere. No numbers necessary. Just draw as much as necessary to make the pattern clear (4 pts).



SCORE: ____________

III. Propagation of Information (25 pts).

A. Conceptual Framework (12 pts).

i. In (approx. 3-5) ***complete*** sentences of English, assess the validity of the following statement:

NOTE

Newton's Laws of Motion apply to the passage of ***information*** from one point in space to another.

Your response must address the way(s) in which the above statement is true AND the way(s) in which the above statement is false. Somewhere in your response, make reference to the concept of "medium".

NOTE

GRADING RUBRIC:

4 pts: MEANINGFUL + COHERENT USE OF TERM MEDIUM

4 pts: ACKNOWLEDGEMENT OF "TRUE" → Newton's LAWS govern the S.H.O. OF EACH PARTICLE AND ARE THE STARTING ASSUMPTION FOR DERIVATION of WAVE EQUATION

4 pts: ACKNOWLEDGEMENT OF "FALSE" → THE PATTERNED S.H.O.'S CREATE PULSES OF INFORMATION (FOR EXAMPLE) WHICH DO NOT HAVE MASS AND DO NOT ACCELERATE.

SCORE: ______________

B. Wave Equation (13 pts)

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\left(\frac{\partial^2 y}{\partial x^2}\right)$$

Picture one little particle in the medium for a wave traveling along the x-axis. The particle is oscillating along the y-axis.

i. What happens to the ***speed*** of the wave along the x-axis if the ***concavity*** of the y-position/x-position graph at the point of the particle is ***quadrupled*** and all else is held constant (4 pts)?

The speed DECREASES by a factor of 2

ii. What happens to the particle's ***acceleration along the y-axis*** if the ***tension*** is quadrupled and all else is held constant (4 pts)?

The particle's acceleration QUADRPULES

iii. Show that ***y = A cos (wt − kx)*** is a solution to the wave equation iff the ***speed*** of the wave is ***w/k*** (5 pts).

$v = \sqrt{\frac{T}{\mu}}$ So $\frac{\partial^2 y}{\partial t^2} = (v^2)\frac{\partial^2 y}{\partial x^2}$

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2 \cos(\omega t - kx)$$

$$\frac{\partial^2 y}{\partial x^2} = -Ak^2 \cos(\omega t - kx)$$

$$\cancel{+A}\omega^2 \cancel{\cos(\omega t - kx)} = v^2(\cancel{+A}k^2 \cancel{\cos(\omega t - kx)})$$

So $v^2 = \frac{\omega^2}{k^2} \longrightarrow \boxed{v = \frac{\omega}{k}}$

SCORE: ______________

IV. Sound Waves (50 pts).

A. Two strings, ***A*** and ***B***, are separated by a distance of 5 meters. They vibrate in phase with each other. They are each fixed at both ends.

Each string has a length of approx. 400 cm.

Each string is fixed at both ends by pegs which exert a tension of approx. 36 Newtons; the 1-dimensional density of each string is approximately .0001 (1×10^{-4}) kg/meter.

i. What is the **frequency** of the ***third harmonic*** for standing waves produced by either of these strings (5 pts)?

$L = \frac{n\lambda}{2}$ Here, $n = 3$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{36\ N}{1\times10^{-4}\ kg/m}} \approx 600\ m/s$$

$$L = \frac{3\lambda_3}{2} \longrightarrow \lambda_3 = \frac{2L}{3} \longrightarrow f_3 = \frac{v}{\lambda_3} \cdot \frac{3}{2L}$$

$$f_3 = \frac{3v}{2L} = \frac{3(600\ m/s)}{2(4)} \rightarrow \boxed{f_3 = 225}$$

ii. Is it possible for either of these strings to produce standing waves that vibrate at a frequency of 1500 Hz? If not, why not? If so, how many nodes will result? (7 pts)?

$f_1 = \frac{225}{3} = 75\ Hz \longrightarrow$ 1500 Hz IS AN INTEGER MULTIPLE of 75 Hz

YES

$\frac{1500\ Hz}{75\ Hz} = 20$

THIS IS THE 20^{th} HARMONIC

SO $\boxed{21\ \text{NODES}}$

SCORE: ______________

iii. Consider the line that runs straight from string A to string B. Somewhere on this line, find the location ***closest to the midpoint between speakers*** at which a receiver could be placed and detect ***no sound*** at all (9 pts).

Assume that the strings vibrate at their fundamental frequency. Also assume that sound travels through ***air*** at approximately 340 m/s.

$f = 75\ Hz$

$\downarrow$

$v = \lambda f$

$\lambda = \frac{v}{f} = \frac{340}{75}$

$\lambda \approx 4.53\ m$

$\Delta \ell = \frac{n\lambda}{2}$

$n \to$ ODD : NODAL LINE

CLOSEST TO MIDPOINT $\Rightarrow n = 1$

So $\Delta \ell = \frac{1 \cdot 4.53}{2}$

$\Delta \ell \approx 2.27$

So THE RECEIVER SHOULD BE PLACED: $\boxed{x \approx 1.14\ m}$ FROM MIDPOINT IN EITHER DIRECTION

B. A car is traveling on a non-windy day.
The car is traveling at a speed of 80 m/s relative to air. It blares a horn.
The car driver hears a frequency of 550 Hz.
You are standing still—relative to air. You observe the car to be *moving toward you* (at 80 m/s relative to air).
Assume that, on this summer day, sound travels at ***348*** m/s relative to air.

i. Find the *wavelength* you measure (5 pts).

$\lambda_R = \frac{348\ m/s - 80\ m/s}{550\ Hz} = \frac{268\ m/s}{550\ Hz} \approx \boxed{.487\ m \approx \lambda_R}$

ii. Find the *frequency* you measure (6 pts).

$f_R = \frac{348\ m/s}{.487\ m} \approx \boxed{714\ Hz = f_R}$

SCORE: ______________

iii. Now, assume that the situation changes:

The car producing the sound has stopped and is now stationary relative to air.

You are now *moving toward the sound source*--at a speed of 80 m/s relative to air. All other quantities are held constant.

a) Find the *wavelength* you measure (5 pts).

$$\lambda_R = \frac{348 \text{ m/s}}{550 \text{ Hz}} \approx \boxed{.633 \text{ m} \approx \lambda}$$

b) Find the *frequency* you measure (7 pts).

$$f = \frac{348 \text{ m/s} + 80 \text{ m/s}}{.633 \text{ m}} \approx 676 \text{ Hz}$$

c) In approximately three complete sentences of English, explain why your answers to (ii) and (iii b), above are ***not*** the same (6 pts).

SCORE: ______________