Final EXAM:

E, M & Radiation

PHYSICS 204

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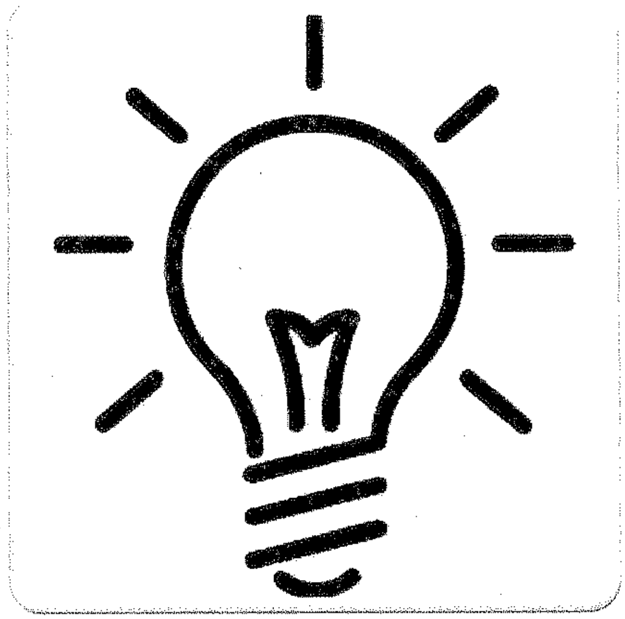
WEDNESDAY, MAY 21, 2014

SECTIONS 1, 2



APPROVED BY YAVERBAUM

Name: SOLUTIONS

Section #: 1

SCORE: ____________

SOME USEFUL RELATIONS:

1) $\oint \vec{E} \cdot \overrightarrow{da} = \frac{q_{(enc)}}{\varepsilon_0}$.

2) $\oint \vec{B} \cdot \overrightarrow{da} = 0$.

3) $\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d}{dt}\int \vec{B} \cdot \overrightarrow{da}$.

4) $\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I_{(enc)} + \mu_0 \varepsilon_0 \frac{d}{dt}\int \vec{E} \cdot \overrightarrow{da}$.

5) $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$.

6) $\vec{F}_E \equiv q\vec{E}$.

7) $\vec{B} = \frac{\mu_0}{4\pi}\frac{q}{r^2}\vec{v} \times \hat{r}$.

8) $\vec{F}_B \equiv q\vec{v} \times \vec{B} = \vec{I}l \times \vec{B}$.

9) $V_a - V_b \equiv \int_a^b \vec{E} \cdot \overrightarrow{dr}$.

10) $C \equiv \frac{Q}{\Delta V}$. (Note: This "C" is capital.)

11) $I \equiv \frac{dq}{dt}$

12) $I = \frac{\Delta V}{R}$.

13) $\mathcal{E} - IR - \frac{Q}{C} = 0$. (Capital "C".)

14) $c \approx 3\ x\ 10^8\ m/s$.

15) $n \equiv \frac{c}{v}$. (Lower-case "c".)

16) $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

17) $\frac{Yd}{L} = \frac{n\lambda}{2}$

18) $sin^2\theta + cos^2\theta = 1$.

SCORE: ________________

19) $\bar{v} \equiv \frac{\Delta x}{\Delta t}$.

20) $\Delta\, t' = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}$. Lower-case "c". Bold-face conclusion.

21) $\sum \vec{F} = m\vec{a}$.

22) $F = -Kx$.

23) $x = Acos(\omega t)$.

24) $\omega = 2\pi f$.

25) $f = \frac{1}{T}$.

26) $KE = \frac{1}{2}mv^2$

27) $PE_{elastic} = \frac{1}{2}Kx^2$

28) $v = \lambda f$.

29) $v = \sqrt{\frac{T}{\mu}}$.

30) $\frac{\partial^2 y}{\partial t^2} = (v^2)\frac{\partial^2 y}{\partial x^2}$.

31) $L = \frac{n\lambda}{2}$.

32) $\Delta l = \frac{n\lambda}{2}$.

34) $\epsilon_0 \approx 8.85\ x\ 10^{-12}\ \frac{C^2}{Nm^2}$.

35) $\mu_0 \approx 1.26 \times 10^{-7}\ \frac{N}{A^2}$.

36) $m_e \approx 9.11\ x\ 10^{-31}$ kg.

37) $q_e \approx 1.60\ x\ 10^{-19}\ C$.

SCORE: ________________

DIRECTIONS

But Sections 1+2 : TOTAL = 150 pts

→ FINAL SCORE =

$115 - \frac{3}{4}(\#\text{WRONG}) =$

The following pages contain ~~110~~ 115 points worth of physics.

$100 - \frac{3}{4}(\text{points lost})$

$+15$

Solve anything and everything you can.

Just know that you will be given a score out of **100**

(which will then be weighted for your final average according to my promise).

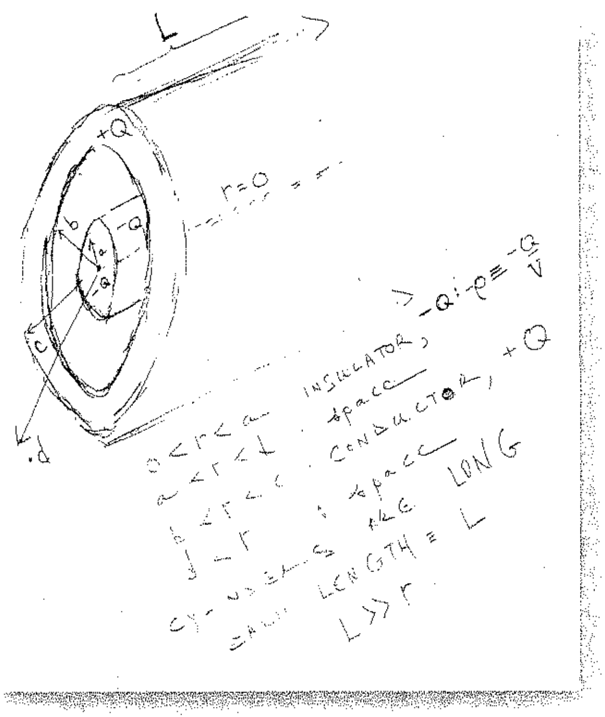
That is, whether you care about these directions or not,

Only your best **100** out of ~~110~~ 115 points will be counted.

May the field be with you.

SCORE: ________________

I. Electrostatics (from a continuous charge distribution) (~~45~~ pts). 50 pts



The above depicts the face-on view (cross-section) of a set of concentric ***CYLINDERS***, EACH of LENGTH ***L.***

The innermost cylinder, radius ***a***, is made of ***insulating*** material.
This ***insulator*** contains a net ***negative*** charge of ***-Q.***
The net ***negative*** charge, ***-Q***, is uniformly distributed throughout the ***cylinder***; $-\rho \equiv \frac{-Q}{V}$.

Outside the ***insulating cylinder***, there is some empty space (vacuum):
Here, there is simply no material to contain charges of any kind.
The space extends from radius ***a*** to a larger radius, ***b***.

Both the ***insulating cylinder*** and the space are surrounded by a ***cylindrical shell***—inner radius, ***b***, outer radius ***c***.

This ***cylindrical shell*** is made of ***conducting material.***
The conducting ***shell*** contains a net ***positive*** charge of ***+Q.***

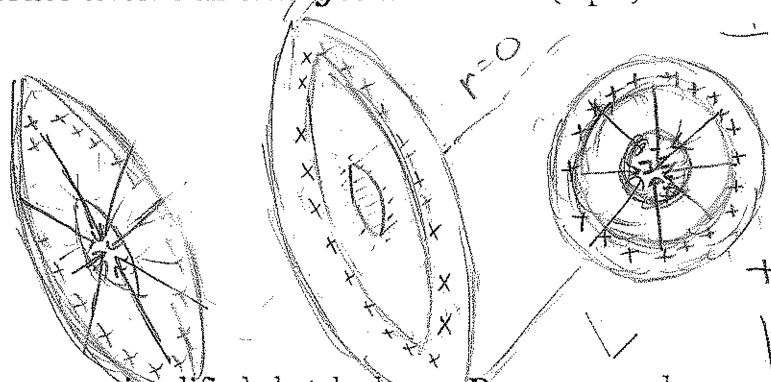
Point ***d*** refers to an arbitrary location of interest fully outside the entire arrangement configuration of cylinder within shell.

This entire configuration has been sitting on a lab table for a long time; it is electrically stable.

SCORE: ______________

SPOKES RADIATING INWARD TO CENTER FROM INNER SURFACE OF CONDUCTOR!

a) Quickly sketch a simplified version of the diagram above. Then draw +'s and −'s to ***indicate where*** all ***charges are*** located (2 pts).



−'s: UNIFORMLY DISTRIBUTED THROUGH VOLUME OF INSULATOR

+'s: UNIFORMLY DISTRIBUTED ON INNER SURFACE OF CONDUCTOR

b) Use your simplified sketch above. ***Draw*** properly representative ***field lines*** anywhere they apply. Draw enough lines to provide a clear sense of ***direction and*** comparative ***magnitude*** (3 pts).

c) Determine the magnitude of the ***electric field*** as a function of position (measured out from the insulator's central axis) for each of the following regions:

i. $0 < r < a$: $E(r) = ?$ (5 pts.)

GAUSSIAN CYLINDER AT r

$$\oint \vec{E}\cdot d\vec{a} = \frac{q(enc)}{\epsilon_0}$$

$$E\oint da = \frac{q(enc)}{\epsilon_0}$$

$$E\cdot 2\pi r L = \frac{q(enc)}{\epsilon_0}$$

$$\frac{q(enc)}{\cancel{\pi} r^2 \cancel{L}} = \frac{Q}{\cancel{\pi} a^2 \cancel{L}} \rightarrow q(enc) = \frac{Qr^2}{a^2}$$

$$E = \frac{1}{2\pi\epsilon_0}\frac{Qr^{\cancel{2}}}{\cancel{r}La^2}$$

$$\boxed{E = \frac{1}{2\pi\epsilon_0}\frac{\lambda r}{a^2}}$$

$$\boxed{\text{Let } \lambda \equiv \frac{Q}{L}}$$

ONLY MAGNITUDE

ii. $a < r < b$: $E(r) = ?$ (5 pts.)

$$\oint \vec{E}\cdot d\vec{a} = \frac{q(enc)}{\epsilon_0}$$

$$E\cdot 2\pi r L = \frac{Q}{\epsilon_0}$$

$$E = \frac{1}{2\pi\epsilon_0}\frac{Q}{Lr} \rightarrow \boxed{E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}}$$

→ MINUS signs not relevant

SCORE: ____________

iii. b < r < c: $E(r)$ = ? (3 pts.)

CONDUCTOR : $\boxed{E = 0 !}$

and, $\therefore$, $q(enc) = 0$

iv. d < r: $E(r)$ = ? (2 pts.)

$$\oint \vec{E} \cdot d\vec{a} = \frac{q(enc)}{\epsilon_0} \longrightarrow q(enc) = 0$$

$$\boxed{E = 0}$$

d) Determine the ***electric potential difference*** as a function of position across each of the following regions. If it helps you, you may assume that $V \to 0$ *as* $r \to \infty$.

i. V(0) – V(a) = ? (5 pts.)

$$V(0) - V(a) \equiv \int_0^a \vec{E} \cdot d\vec{r} = \int_0^a -\frac{1}{2\pi\epsilon_0} \frac{\lambda r}{a^2} \cdot dr = -\frac{1}{2\pi\epsilon_0} \frac{\lambda}{a^2} \int_0^a r \, dr$$

$$\Delta V = -\frac{1}{2\pi\epsilon_0} \frac{\lambda}{a^2} \frac{r^2}{2} \Big]_0^a = -\frac{1}{4\pi\epsilon_0} \frac{\lambda}{\cancel{a^2}} \cdot \cancel{a^2} = \boxed{-\frac{\lambda}{4\pi\epsilon_0} = V(0) - V(a)}$$

ii. V(a) – (b) = ? (5 pts.)

$$V(a) - V(b) \equiv \int_a^b \vec{E} \cdot d\vec{r} = \int_a^b -\frac{1}{2\pi\epsilon_0} \frac{\lambda}{r} dr = \frac{-\lambda}{2\pi\epsilon_0} \int_a^b \frac{1}{r} dr$$

$$= -\frac{\lambda}{2\pi\epsilon_0} \left(\ln \frac{b}{a} \right) \quad V(a) - V(b) = \frac{\lambda}{2\pi\epsilon_0} \left(\ln \left[\frac{a}{b} \right] \right)$$

SCORE: ______________

iii. V(b) – V(c) = ? (3 pts.) $E = 0$

$$\boxed{V(b) - V(c) = 0}$$

CONDUCTORS ARE EQUIPOTENTIAL REGIONS!

iv. V(c) – V(d) = ? (2 pts.)

AGAIN, $E = 0$

$$\boxed{V(c) - V(d) = 0}$$

v. V(o) – V(d) = ? (3 pts.) $V(o) - V(d) \equiv [V(o) - V(a)] + [V(a) - V(b)]$

$$+ [V(b) - V(c)] + [V(c) - V(d)] = -\frac{\lambda}{4\pi\epsilon_0}\left(1 + 2\ln\left[\frac{b}{a}\right]\right)$$

$\Delta V = \frac{\lambda}{4\pi\epsilon_0}\left[2\ln\left(\frac{a}{b}\right) - 1\right]$

vi. V(o) – $V(\infty)$ = ? (2 pts.)

$V(d) - V(\infty) = 0$

$$\text{so } V(o) - V(\infty) = -\frac{\lambda}{4\pi\epsilon_0}\left(1 + 2\ln\left[\frac{b}{a}\right]\right)$$

e) Find ***C***, the ***capacitance*** of this physical arrangement:
Specifically, find ***C*** between ***r = o*** and ***r = d*** (3 pts).

(We say ***d***, rather than **c**, just to make absolutely certain that we account for every possible bit of material, charge and space included in this arrangement.)

Your answer will be expressed in terms of given and fundamental constants.

$$C \equiv \frac{Q}{\frac{\lambda}{4\pi\epsilon_0}\left(1 + 2\ln\left[\frac{b}{a}\right]\right)} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\epsilon_0 L}\left(1 + 2\ln\left[\frac{b}{a}\right]\right)}$$

$$\boxed{C = \frac{4\pi\epsilon_0 L}{\left(1 + 2\ln\left[\frac{b}{a}\right]\right)}}$$

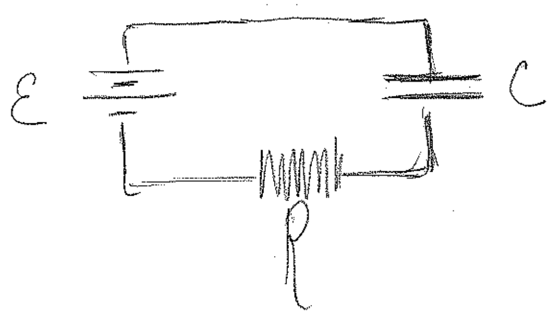
SCORE: ______________

Now assume the following:

a = 3 cm.
b = 8 cm.
c = 10 cm.
L = 10 cm.

The center of the ***insulating cylinder*** is connected by wire to a ***50 ohm resistor***. The other end of this resistor is connected to the negative terminal of a ***battery***. The positive terminal of the battery is connected to the ***conducting shell***.

f) Draw a circuit diagram for this new situation. You may use the standard symbol for capacitor even though this capacitor happens not to be composed of plates (2 pts).



g) From the moment the wires are all connected, current flows. As it flows, its rate of flow continuously slows down.

From the moment the wires are all connected, how much time will pass until this current decays to 1 e^{th} (approx.. $\frac{1}{2.71...}$) of its initial value (5 pts)?

THIS ANSWER WILL BE NUMERICAL – and should be expressed with appropriate units.

$$I = I_0 e^{-\frac{t}{RC}} \longrightarrow \frac{\cancel{I_0}}{e} = \cancel{I_0} e^{-t/RC}$$

$$\frac{1}{e} = e^{-t/RC} \longrightarrow e^{-1} = e^{-t/RC}$$

$$-1 = -t/RC \longrightarrow 1 = \frac{t}{RC}$$

$$= \frac{4\pi\epsilon_0 L}{(1+2\ln[\frac{b}{a}])}$$

$$= \frac{4\pi(8.85\times10^{-12})(.10)}{(1+2\ln[\frac{8}{3}])}$$

$\therefore t = RC$ (of course!)

$$\boxed{t \approx 1.88\times10^{-10}\ sec}$$

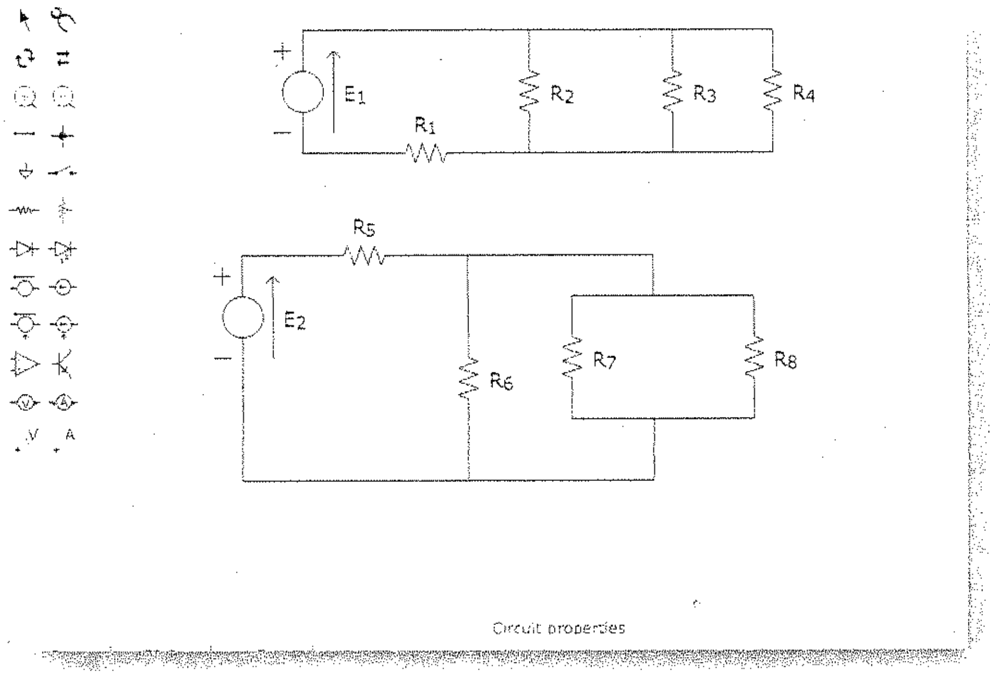
$$C \approx 3.76\times10^{-12}\ F$$

$$C = 3.76\times10^{-6}\ \mu F \longrightarrow \text{so } t \approx (50\,\Omega)(3.76\times10^{-12}\ F) \approx 1.88\times10^{-10}\ sec$$

SCORE: ______________

II. A CIRCUIT (30 pts).

A. Examine the following two circuits (Values given under "Circuit Properties".)



E1	E2	15 Volts
R1	R5	175 Ohms
R2	R6	50 Ohms
R3	R7	60 Ohms
R4	R8	300 Ohms

Is the current flowing through the BATTERY (i.e. "battery loop"):

Greater in the first circuit,
Greater in the second circuit
OR
The same in both? In a complete English sentence or two, explain! (6 pts.)

IDENTICAL in BOTH → $R_3 \parallel$ with R_4
→ $R_7 \parallel$ with R_8
all else IDENTICAL

SCORE: ______________

B. For JUST THE FIRST of the two circuits, determine:

i. The ***current*** flowing through each and every RESISTOR (12 pts: 3 pts each).

I (R1): $\frac{1}{R_{34}} = \frac{1}{R_3} + \frac{1}{R_4} \rightarrow \frac{1}{R_{34}} = \frac{1}{60} + \frac{1}{300} = \frac{5}{300} + \frac{1}{300}$

$\frac{1}{R_{34}} = \frac{6}{300} = 50 \rightarrow \frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_{34}} = \frac{1}{50} + \frac{1}{50} \rightarrow R_{234} = 25$

$R_{1234} = 175 + 25 = 200\,\Omega \longrightarrow I_1 = I_B = \frac{15\,V}{200\,\Omega} = \boxed{.075\,A = 75\,mA =}$

I (R2):

EVERY CHARGE MUST FLOW either THROUGH R_2 or R_{34}
$50\,\Omega$ $50\,\Omega$
so it splits EVENLY:

$I_2 = \frac{.075\,A}{2} = \boxed{37.5\,mA = I}$

EVERY CHARGE NOT through R_2 flows either through R_3 or R_4

I (R3):

$\frac{5}{6}(37.5\,mA) = \boxed{I_3 = 31.25\,mA}$

$\frac{1}{6}(37.5\,mA) = I_4 = 6.25\,mA$

I (R4):

$\boxed{I_4 = 6.25\,mA}$

 SCORE: ______________

ii. The ***potential difference*** across each and every RESISTOR (12 pts: 3 pts each).

ΔV (R1): $\Delta V(R_1) = I_1 R_1 = (.075\ A)(175\ \Omega)$

$\Delta V(R_1) = 13.125\ V$

ΔV (R2): $\Delta V(2) = I_2 R_2$

$\Delta V(R_2) = (.0375\ A)(50\ \Omega)$
$= 1.875\ V$

$13.125\ V + 1.875\ V$
$= 15$ Volts!

Energy Conservation!

ΔV (R3):

MUST BE 1.875 V. CHECK: $\Delta V(R_3) = (.03125\ A)(60\ \Omega)$

$\boxed{\Delta V(R_3) = 1.875\ V}$ ✓

ΔV (R4):

MUST BE 1.875 V. CHECK: $\Delta V(R_4) = (.00625\ A)(300\ \Omega)$

$\boxed{\Delta V(R_4) = 1.875\ V}$ ✓

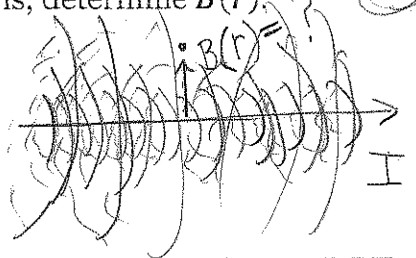
SCORE: ________________

III. **B-Fields** (20 pts).

A. Use AMPERE's LAW in order to determine the ***magnetic field*** as a function of ***position*** near a LONG, STRAIGHT CURRENT-CARRYING WIRE, ***I*** (10 pts).

DRAW AN APPROPRIATE and fully labeled DIAGRAM!

That is, determine $\vec{B}(r)$.



(+2) DRAW AN AMPERIAN (CLOSED) CIRCLE, CENTERED AT I, RADIUS r.

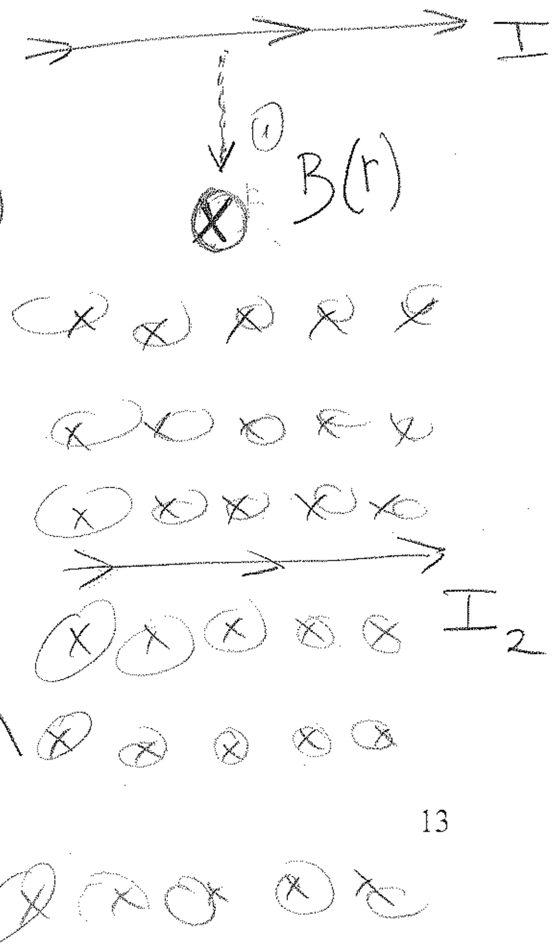
(+2) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{(enc)}$

$B\oint dl = \mu_0 I_{(enc)} \rightarrow B\cdot 2\pi r = \mu_0 I_{en}$ (+2)

(+2) $B = \dfrac{\mu_0}{2\pi}\dfrac{I_{enc}}{r}$ and $I_{enc} = I$ so (+2)

$$\boxed{B = \frac{\mu_0}{2\pi}\frac{I}{r}}$$

B. Use the DIRECTIONS demanded by the ***Biot-Savart*** and ***Lorentz Force Laws*** to explain a fundamental finding (from 1820): ***WHY*** do two ***parallel currents attract*** (or why two anti-parallel currents repel) (10 pts)? +2 (+2)



(1)

$d\vec{B} = \dfrac{\mu_0}{2\pi} \dfrac{I\, d\vec{l} \times \hat{r}}{r^2}$

$d\hat{B} = d\vec{l} \times \hat{r}$

FIELD BELOW right-pointing CURRENT POINTS IN

SO NOW I_2 IS SUBJECT TO FORCE FROM INWARD-pointing FIELD

(1) (+2) (+2)

$\hat{F} = \vec{l} \times \vec{B}$:

FORCE ON right-pointing I_2 FROM IN-pointing B POINT UP

13

SCORE: ____________

$\vec{F} = I\vec{l} \times \vec{B}$

FORCE ON I_2 POINTS

IV. Electromagnetic Radiation (15 pts).

FIND & CORRECT *ANY THREE IMPORTANT MISTAKES* IN THE FOLLOWING ARGUMENT. (There are more than 3.)

THE MORE SPECIFICALLY YOU CAN EXPLAIN THE MEANING and/or SIGNIFICANCE OF THE MISTAKES, the more points you will gain!

With the inclusion of Maxwell's displacement current, the corrected Ampere's Law becomes:

(1)

$$\oint \cancel{\vec{E}}\ \vec{B} \cdot \overrightarrow{dl} = \mu_0 I_{(enc)} + \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot \overrightarrow{da}.$$

Just like a ***change*** in ***magnetic flux***
through some area
induces an ***electric field***
through the ~~***closed surface***~~
bounding that area

(2) PATH: THE BOUNDARY OF AN n-DIMENSIONAL REGION MUST BE n-1 DIM.!

THIS IS Ampere's LAW: IT ESTABLISHES THE MAGNETIC FIELD FROM AN ELECTRIC CURRENT;

$\oint \vec{E} \cdot \vec{dl}$ ≡ ELECTRIC POTENTIAL = VOLTAGE which

$\oint$ IS 0 unless INDUCED!)

it's also (and equally) true that

a ***change*** in ***electric flux***
through some ~~volume~~
induces an ***magnetic field***
through the ***closed path***
bounding that area

(3) AREA!

$\Phi_E \equiv \int \vec{E} \cdot \vec{da} \equiv \vec{E}$ THROUGH AREA!

Look at what must be true in the free space—far away from any pieces of charge or current. There, the four Maxwell's Equations become:

1) $\oint \vec{E} \cdot \overrightarrow{da} = 0.$

2) $\oint \vec{B} \cdot \cancel{\overrightarrow{dl}}\ \vec{da} = 0.$

(4) No magnetic monopole (no all lines are closed loops) out = in otherwise, contradict AMPERE

3) $\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d}{dt} \int \vec{B} \cdot \overrightarrow{da}.$

(5) the induced potential must oppose the change in Φ_B otherwise

4) $\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \cancel{\vec{B}}\ E \cdot \overrightarrow{da}.$

(5) DISPLACEMENT CURRENT

ENERGY CONSERVATION VIOLATED! $\rightarrow \frac{dB}{dt} \Rightarrow \oint \vec{E} \cdot \vec{dl} \Rightarrow I \Rightarrow \frac{dI}{dt} \Rightarrow \frac{dB}{dt} \ldots \rightarrow \infty$

SCORE: ______________

Rearranging ("decoupling") the equations so that we can look at electric fields on their own and magnetic fields on their own (see following pages for method),

We find:

$$\frac{\partial^2}{\partial A^2}\vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2}\vec{E}$$

and

$$\frac{\partial^2}{\partial A^2}\vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2}\vec{B}.$$

Look familiar?

6 WAVE EQUATION!

Remember ripples on a string?

$$\frac{\partial^2}{\partial x^2} y = \frac{1}{v^2}\frac{\partial^2}{\cancel{\partial y^2}\ \partial t^2} y$$

The mutually inducing perpendicular fields travel through space as an WAVE for which

$$*** \quad v = \frac{1}{\sqrt{G_m m \varepsilon_0}} = c! \quad ***$$

7 must be the constant of proportionality btw 2nd/ space derivative + 2nd time derivative.

$$V = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

*And, thus Dear Galileo, the head-***light*** from an express train travels at *c*
relative to observers on that speeding train, yet also
travels at (that same) *c*
relative to observers on the platform.

So follow that train of thought – it's definitely an A train;
then have a great summer and, soon enough, a great time in P-Chem!

It's been my honor and pleasure to share this spacetime interval with you.

SCORE: ________________