Final Exam (3.14)

Charge, Flow, Field, Flux & Lux

Physics 204, Exam Sections 01.L1, 01.L2

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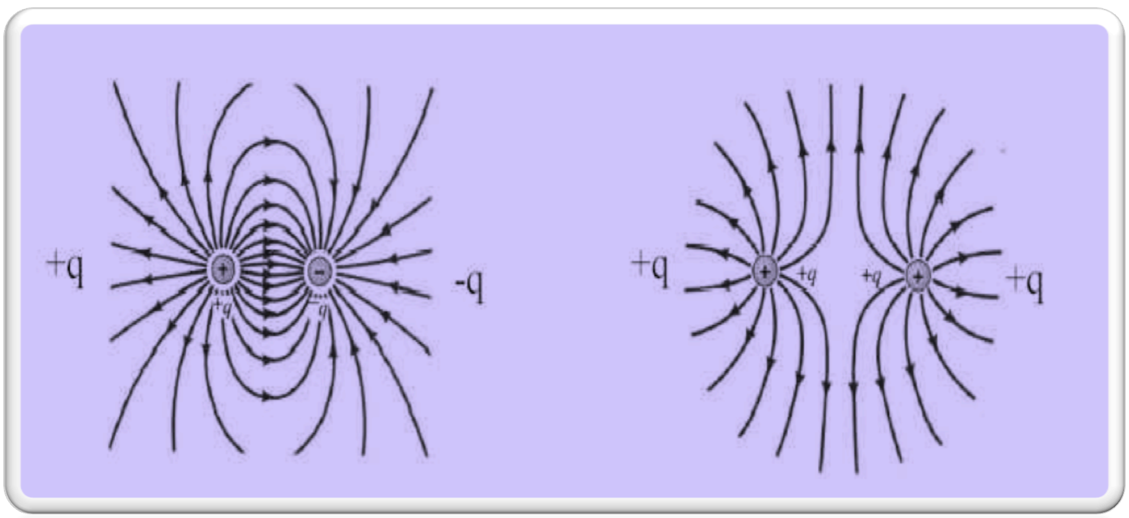
DUE 11:59 pm, Friday, July 17, 2015



Name: __SOLUTIONS________

Section #: __42__________

SOME USEFUL RELATIONS:

1) $\oint \vec{E} \cdot \overrightarrow{da} = \frac{q_{(enc)}}{\varepsilon_0}$.

2) $\oint \vec{B} \cdot \overrightarrow{da} = 0$.

3) $\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d}{dt} \int \vec{B} \cdot \overrightarrow{da}$.

4) $\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I_{(enc)} + \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot \overrightarrow{da}$.

5) $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$.

6) $\vec{F}_E \equiv q\vec{E}$.

7) $\vec{B} = \frac{\mu_0}{4\pi} \frac{q}{r^2} \vec{v} \times \hat{r}$.

8) $\vec{F}_B \equiv q\vec{v} \times \vec{B} = \vec{I} l \times \vec{B}$.

9) $V_a - V_b \equiv \int_a^b \vec{E} \cdot \overrightarrow{dr}$.

10) $C \equiv \frac{Q}{\Delta V}$. (Note: This "C" is capital.)

11) $I \equiv \frac{dq}{dt}$

12) $I = \frac{\Delta V}{R}$.

13) $\mathcal{E} - IR - \frac{Q}{C} = 0$. (Capital "C".)

14) $c \approx 3\ x\ 10^8\ m/s$.

15) $n \equiv \frac{c}{v}$. (Lower-case "c".)

16) $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

17) $\frac{Yd}{L} = \frac{n\lambda}{2}$

$$18)\ sin^2\theta + \ cos^2\theta = 1.$$

$$19)\ \bar{v} \equiv \frac{\Delta x}{\Delta t}.$$

$$20)\ \sum \vec{F} = m\vec{a}.$$

$$21)\ F = -Kx.$$

$$22)\ x = Acos(\omega t + \phi).$$

$$23)\ \omega = 2\pi f.$$

$$24)\ f = \frac{1}{T}.$$

$$25)\ KE = \frac{1}{2}mv^2$$

$$26)\ PE_{elastic} = \frac{1}{2}Kx^2$$

$$27)\ v = \lambda f.$$

$$28)\ v = \sqrt{\frac{T}{\mu}}.$$

$$29)\ \frac{\partial^2 y}{\partial t^2} = (v^2)\frac{\partial^2 y}{\partial x^2}.$$

$$30)\ \epsilon_0 \approx 8.85\ x\ 10^{-12}\ \frac{C^2}{Nm^2}.$$

$$31)\ \mu_0 \approx 1.26 \times 10^{-7}\ \frac{N}{A^2}.$$

$$32)\ m_e \approx 9.11\ x\ 10^{-31}\ \text{kg}.$$

$$33)\ m_{(PROTON)} \approx m_{(NEUTRON)} \approx 1.67\ x\ 10^{-27}\ \text{kg}.$$

$$34)\ q_e \approx 1.60\ x\ 10^{-19}\ C.$$

DIRECTIONS

Open-Book; Open-Text; Open-Web;
Open-Colleagues, Collaborators, Calculators & Conquistadors:
Your understanding may come from anywhere and everywhere,
but it must end up as your understanding—

demonstrated by prodigious explanations,
meticulous diagrams,
Thorough and complete processes of thought

And an explicit recognition that the grading of these last two exams
will be conducted with unrestrained scrutiny and fussiness:
Your job is **not** simply to demonstrate that you are in possession
of correct answers. For that, *Google* can have an A with my blessings.
Your job is to demonstrate comprehension of reasoning and appreciation of sense.

SO…

Your exam will ultimately consist of FIVE (5) problems,
as follows:

YOU *MUST* SOLVE ***EVERY ONE*** OF THE FIRST THREE PROBLEMS
(All of I – III Mandatory),

Then, you must CHOOSE EITHER
Problem IV OR Problem V,

Then, you must answer ALL of the LAST PROBLEM
(Problem VI Mandatory).

MAY THE FIELD BE WITH YOU.

I. E-Fields from ***Point Charges*** (20 pts).

Two ***point-charges*** of differing magnitudes and are held stationary in an enormously spacious x-y plane.

A researcher places an instrument called a 'field detector' at the point (-5,+12). She is interested in measuring the electric field at that precise location.

The two point charges are as follows:

Name	Charge	x-Coordinate	y-Coordinate	Ordered Pair
Q_1	$+5 \times 10^{-10}$ Coulombs	+5 meters	0 meters	(5,0)
Q_2	-12 $\times 10^{-10}$ Coulombs	0 meters	-12 meters	(0,-12)

Location of Interest: (-5,+12)

Note: All coordinates are measured and given in **meters** (not centimeters); similarly, the (enormous) charge magnitudes are in whole **Coulombs** (not micro-Coulombs).

Also Note: If you wish, you are permitted and encouraged to approximate the electrostatic constant as:

$$K_e \approx 10 \times 10^9 \approx 1 \times 10^{10} \frac{Nm^2}{C^2}.$$

* * *

a) Draw a neat and clear sketch of the situation, as you understand it. Your sketch must express a clear decision as to which directions are designated by + and – on each axis (3 pts).

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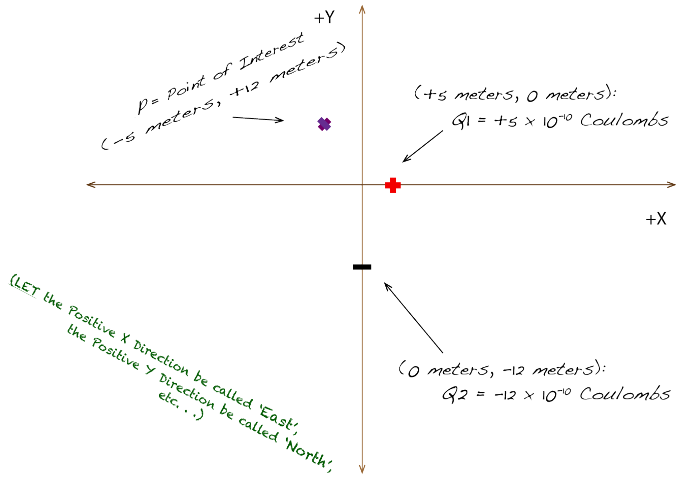
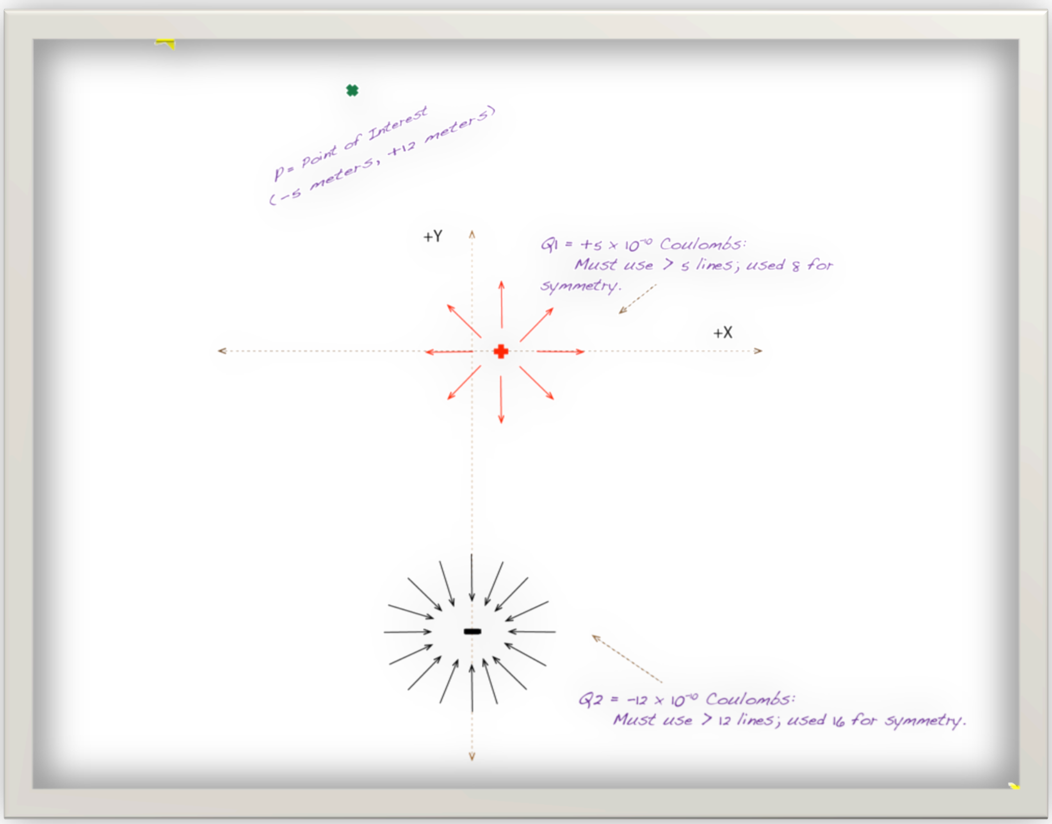


b) For *each* of the two individual point charges, draw an approximate field line diagram – each drawn as it would look IF the other charge did NOT exist. For BOTH, however, obey the following convention: For every 1×10^{-10} Coulombs of strength that generates a field, at least one more field line should appear in the field line diagram (3 pts).



c) Compute the Electric Field as measured at the Point of Interest (-5,+12).

That is:

i. In Newtons/Coulomb, determine the ***electrostatic field*** **magnitude** at this location of interest (5 pts).

X --> ...

$$\sum E_x = K_e\left[\left(\frac{Q_1}{r_1^2}\cdot\frac{x_1}{r_1}\right)+\left(\frac{Q_2}{r_2^2}\cdot\frac{x_2}{r_2}\right)\right]$$

$$\sum E_x = K_e\left[\left(\frac{5\times10^{-10}}{244}\cdot\frac{-10}{\sqrt{244}}\right)+\left(\frac{12\times10^{-10}}{601}\cdot\frac{5}{\sqrt{601}}\right)\right]$$

$$\sum E_x = K_e(1\times10^{-10})\left[\left(\frac{5}{244}\cdot\frac{-10}{\sqrt{244}}\right)+\left(\frac{12}{601}\cdot\frac{5}{\sqrt{601}}\right)\right]$$

$$\sum E_x \approx \cancel{(1\times10^{10})}\,\cancel{(1\times10^{-10})}\left[\left(\frac{5}{244}\cdot\frac{-10}{\sqrt{244}}\right)+\left(\frac{12}{601}\cdot\frac{5}{\sqrt{601}}\right)\right]$$

$$\sum E_x \approx \left[\left(\frac{-50}{3.81\times10^{3}}\right)+\left(\frac{60}{1.47\times10^{4}}\right)\right]$$

$$\approx \left[(-1.31\times10^{-2})+(4.08\times10^{-3})\right]$$

$$\sum E_x \approx -9.02\times10^{-3}\ N/C$$

Y --> ...

$$\sum E_y = K_e\left[\left(\frac{Q_1}{r_1^2}\cdot\frac{y_1}{r_1}\right)+\left(\frac{Q_2}{r_2^2}\cdot\frac{y_2}{r_2}\right)\right]$$

$$\sum E_y = \cancel{K_e}/\cancel{K_e}\left[\left(\frac{5}{244}\cdot\frac{12}{\sqrt{244}}\right)+\left(\frac{12}{601}\cdot\frac{-24}{\sqrt{601}}\right)\right]$$

$$\sum E_y \approx \left[\left(\frac{60}{3.81\times10^{3}}\right)-\left(\frac{288}{1.47\times10^{4}}\right)\right]$$

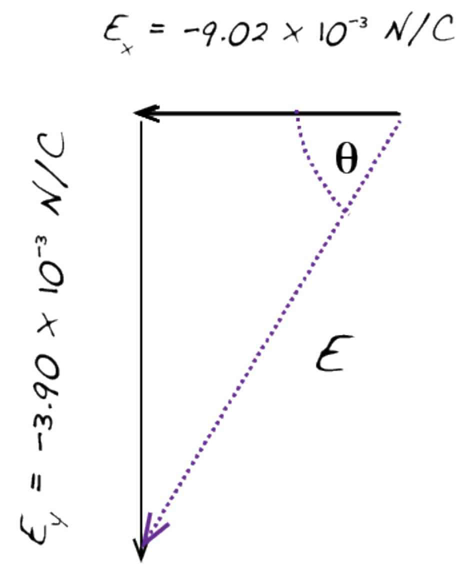
$$\sum E_y \approx \left[(1.57\times10^{-2})-(1.96\times10^{-2})\right]$$

$$\sum E_y \approx \left[(1.57\times10^{-2})-(1.96\times10^{-2})\right]$$

$$\sum E_y \approx -3.90\times10^{-3}\ N/C$$

$$\vec{\mathbf{E}} \equiv \sum E_x + \sum E_y$$

Continued on next page . . . →



$$\vec{\mathbf{E}} \equiv \sum E_x + \sum E_y$$

$$E \equiv \|\vec{\mathbf{E}}\| \approx \sqrt{(9.02 \times 10^{-3})^2 + (-3.90 \times 10^{-3})^2}$$

$$\boxed{E \approx 9.83 \times 10^{-3}\ N/C}$$

ii. Using degrees **and** points of the compass (such as "20 degrees South-West"), express the **direction** of the ***electrostatic field*** at this Point of Interest (-5,+12) (3 pts).

$$\theta \approx \tan^{-1}\left(\frac{3.90 \times 10^{-3}}{9.02 \times 10^{-3}}\right)$$

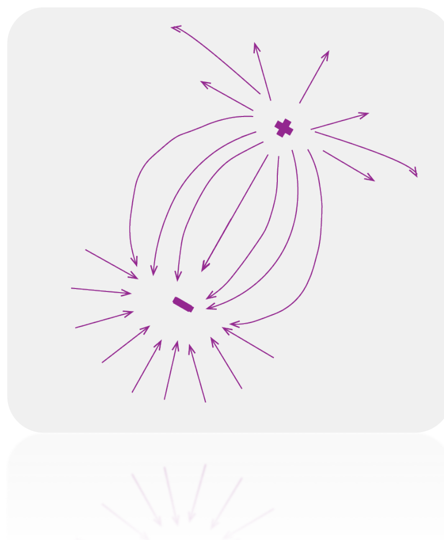
$$\theta \approx 1.03\ radians\ NW\ \ OR\ldots$$

$$\boxed{\theta \approx \ \ 23.4^\circ\ South - West}$$

So,

$$\boxed{\vec{E} \approx 9.83 \times 10^{-3}\ N/C\ at\ 23.4^\circ\ South - West}$$

d) Staying approximately consistent with whatever scale, style (etc.) was involved in your diagrams for (b), above, try now to bring the two graphics together into one visual ***superposition*** of the situation: That is, draw one field line diagram for the net influence exerted by this pair of point charges. In your mind, start bringing the two pictures increasingly close together… Since lines cannot, however, cross nor disappear, allow denser regions of lines from one charge to gently and smoothly 'bend back' (or forward) the lines from another – doing as much as you can to preserve simplicity and symmetry where applicable (3 pts).



e) Assume that a somehow isolated and highly condensed Helium Nucleus (consisting two protons, two neutrons and no other measurably significant entities) is introduced and held gently at the point (-5,12). Assume, further, that any uncharged particles trapped in that nucleus simply 'go along for the ride' whenever the charged particles are motivated to travel. The nucleus is then released. Compute the nucleus 's initial instantaneous acceleration. Provide precise **magnitude** AND **direction** (3 pts).

PLEASE SEE NEXT PAGE
for solution sample . . .

$$\vec{\mathbf{F}}_e \equiv q\vec{\mathbf{E}}.$$

$$Here, q = 2\ protons.$$
$$\vec{\mathbf{F}}_e \approx (2 \times 1.60 \times 10^{-19} C)\, \vec{\mathbf{E}}$$
$$\vec{\mathbf{F}}_e \approx (3.2 \times 10^{-19} C)\,(9.83 \times 10^{-3} N/C)$$
$$\vec{\mathbf{F}}_e \approx 3.15 \times 10^{-21} N \text{ at} \sim 23.4^\circ \text{ South} - \text{West}$$

$$\sum \vec{\mathbf{F}} = m\vec{a}.$$

Assuming that this electrostatic field

is isolated from all other influences, then

the force exerted by the field is the only one

acting on the helium nucleus...

$$\vec{\mathbf{F}}_e = m\vec{a}$$
$$a = \| \vec{a} \| = \frac{F_e}{m_{hn}}$$
$$m_{hn} \equiv mass\ of\ helium\ nucleus \approx 2p + 2n \approx 4p$$
$$m_{hn} \approx 4\,(1.67 \times 10^{-27} kg)$$
$$a \approx \frac{(3.15 \times 10^{-21} N)}{(6.68 \times 10^{-27} kg)}$$

At a given point, any positive charge is accelerated in the SAME direction as the

(tangent line to the) field line at that point:

$$\boxed{\begin{array}{l} a \approx 4.71 \times 10^5 m/s^2 \\ \quad \text{at} \sim 23^\circ\ S - W. \end{array}}$$

II. Gauss's Law (25 pts).

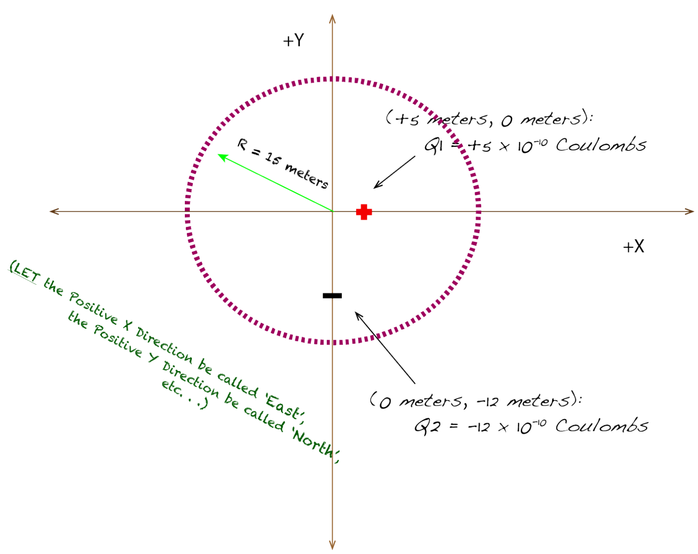
A huge but imaginary sphere is drawn with its center at the origin of a coordinate system, such as that used in Problem I, above.
The radius of the sphere is ***r = 15 meters***.

Two point charges have been sitting within a 15 meter radius of the origin; they continue to sit there.

One charge has a magnitude of +5 x 10-10 Coulombs;
the other charge has a magnitude of -12 x 10-10 Coulombs.
Nothing else exists within 15 meters of the origin.

a) *Draw this situation* as you understand it (2 pts).

(This is not meant to be a trick; the drawing will just help clarify.)



b) In $\frac{Newtons \cdot Meters^2}{Coulombs}$, find the total amount of ***electric flux*** that passes through the surface of this imaginary sphere (2 pts).

Your answer should be a number, expressed in the units mentioned directly above.

Definition of Electrostatic FLUX:

$\Phi_E \equiv \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}.$

Gauss's Law regarding Electrostatic FLUX:

$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{(enc)}}{\varepsilon_0}$

(The electrostatic flux through ANY closed surface
is always and simply the total magnitude of the charge
enclosed by that surface - -divided by a constant
(known as the 'permittivity of free space') - -
entirely independent of the size or shape
of the closed surface,
entirely independent of the shape
of the enclosed charge
and
entirely independent of ANYcharges located anywhere
OUTSIDE the closed surface.
(!)
So, the answer is simply:

$$\frac{(5.00 \times 10^{-10}\, Coulombs) - (12.0 \times 10^{-10}\, Coulombs)}{\varepsilon_0}$$

$$\approx \frac{-7.00 \times 10^{-10}\, Coulombs}{8.85 \times 10^{-12}\, {C^2}/{N \cdot m^2}}$$

$$\boxed{\Phi_E = q_{(enc)} \approx -79.1\, Nm^2/C!}$$

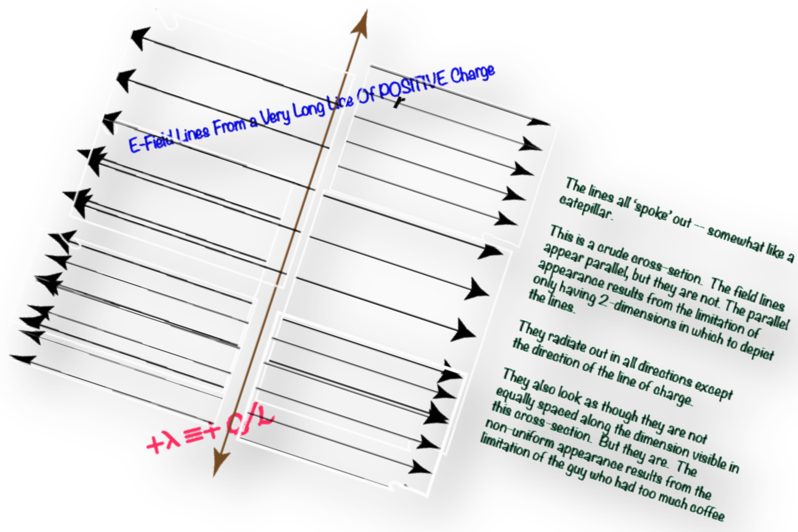
c) The two point charges from Problem I are replaced with an extremely long and straight ***wire*** (i.e.: LINE) that is net-positively charged; this straight line wire stretches through the points (+5,0) and (0,-12,) and beyond.

The charge density in the wire is constant and expressed as follows, where ***L*** refers to length measured in meters and ***Q*** is charge measured in Coulombs:

$$\boldsymbol{\lambda} \equiv \frac{+Q}{L}.$$

Your first large goal, after a series of smaller 'build-up' questions to follow on the next page, will be to compute the electric field as a function of perpendicular distance, ***r****, from this line of charge. . .* (continued on next page) . . .

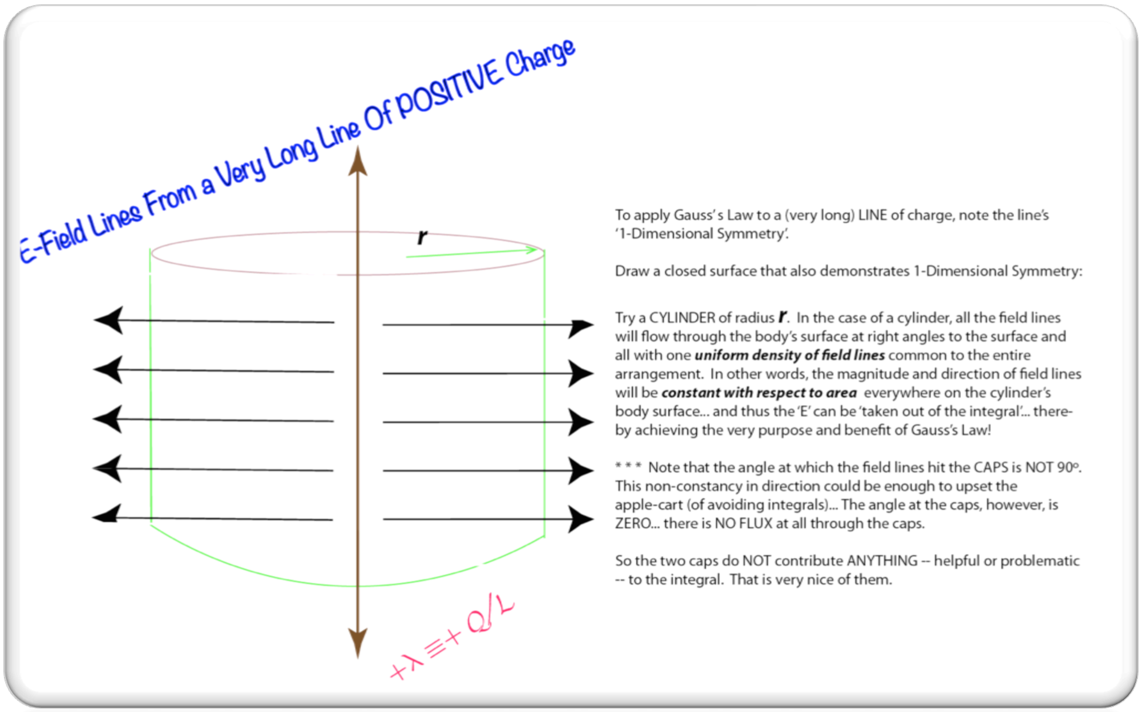
i. Draw the field line diagram produced by this long line of positive charge (2 pts).



ii. In a sentence or two of English, explain why the sphere drawn above will not be the most convenient shape for computing ***electric field*** at ***r*** (3 pts).

Gauss's Law is true for ANY closed surface, so a sphere CAN absolutely be drawn around a vertically oriented continuous line of charge. The total flux through the surface of that sphere WILL be the total charge ($-\lambda L$) enclosed by that sphere - despite the fact that a sphere produces field lines that are symmetric in all directions with NO exception ("0-dimensional symmetry") while a line is symmetric in all directions save ONE (its own axis - hence "1-dimensional symmetry"). Truth is not the problem. The problem is that this difference in pattern (between a line and a sphere) means that the lines spoking from the line of charge will all 'flow' (***flux***) through the sphere surface at different angles and in densities that will differ from region to region. The magnitude and the direction of the field lines, that is, will NOT BE A CONSTANT through the surface of that sphere. So the E-Field term in the Flux Integral CANNOT be 'taken out' of the integral and we actually have to compute an integral. And not an easy one, to say the least. A central purpose of Gauss's Law is to avoid cumbersome or intractable integral calculations by thinking conceptually.... Physically instead. To trade math for physics. If we draw a sphere around the line of charge, Gauss's Law is still right, but it just isn't helpful. We want to set up an integral that we do NOT actually have to do.

iii. Choose and draw a Gaussian (closed) surface surrounding some portion of the line—a surface that will best help you compute the field at $\boldsymbol{r}$ (2 pts).|



iv. Starting with Gauss's Law and proceeding through as many clear and verbally explained steps as possible,

find $\boldsymbol{E}$ as a function of (perpendicular distance) $\boldsymbol{r}$
(given $\boldsymbol{\lambda}, \boldsymbol{\varepsilon_0}$)
from this line of charge (5 pts).

This answer will not be a number.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{(enc)}}{\varepsilon_0}$$

$$E \oint dA = \frac{q_{(enc)}}{\varepsilon_0}$$

(See explanation in 2 (b) ii, above.)

NOTE: On the one hand, if you cannot legitmately make this simplification,

then Gauss's Law does you no good.

Applied properly, the E will ALWAYS 'come out' of the integral.

On the other hand, you cannot simply skip over all the drawing

and over the CHOOSING of the most convenient closed ('Gaussian') surface to draw:

It might well seem as though all the drawing and choosing is all nonsense - -

that we should just write down $EA = \dfrac{q(enc)}{\varepsilon_0}$ and get on with it

since it's apparently going to be true every time.

. . . But Gauss was no fool. Neither be you! Should you wish to drain the bathwaters of thick integral computation, then clutch tightly the babies of well-chosen surface and meaningful integral set-up! . . . →

On the contrary: The drawing and choosing is the heart of the method.

Without drawing and choosing, you don't know WHAT SURFACE AREA is being used

(supposedly by you!) and thus you DO NOT KNOW by what expression you will divide FLUX

in order to get FIELD - -

Determination of the FIELD WAS AND IS THE GOAL of considering Flux. So... back to this calculation

of the E - Field at a distance **r** from a very long line of charge, . . .

$$\ldots E\oint dA = \frac{q_{(enc)}}{\varepsilon_0}$$

$$EA = \frac{q_{(enc)}}{\varepsilon_0}$$

In this particular example/application, therefore,

'area' refers to the surface area of a CYLINDER (radius r, length L):

Surface Area(cylinder) = 2πrL.

$$E \cdot 2\pi rL = \frac{q_{(enc)}}{\varepsilon_0}$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q}{L}\frac{1}{r}\ldots by\ definition, then, \ldots$$

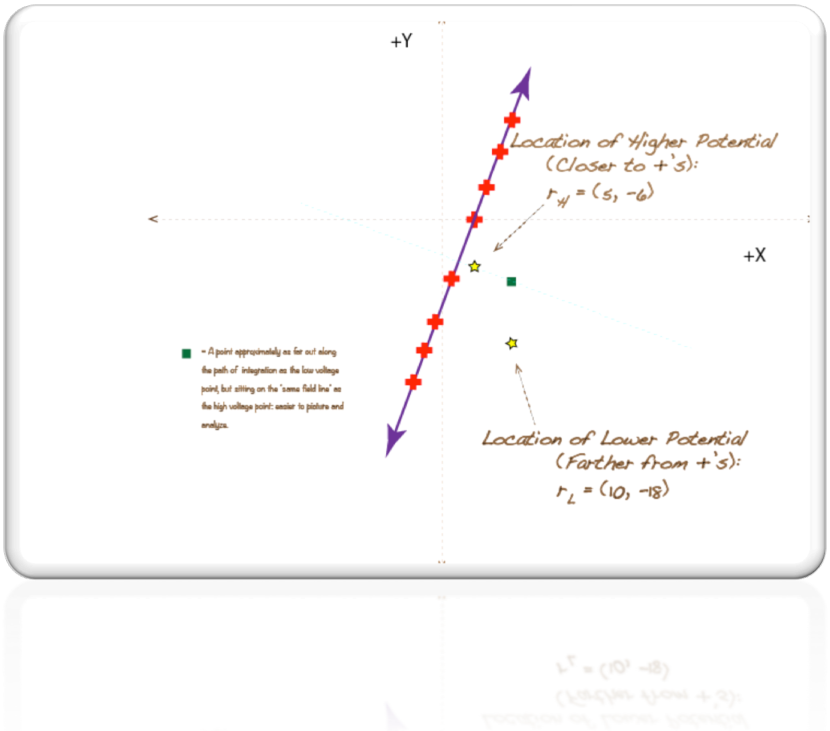
$$\boxed{E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}!}$$

v. Now assume that $\lambda \equiv 5\frac{Coulombs}{meter}$.

In VOLTS, Find the ***Electric Potential Difference*** ('Voltage'!) that exists
(as long as that line remains net-charged)
between the point **(5,-6)** and the point **(10,-18)**.

This is not a joke. Nor a trick.
Hint: Recall and deploy the definition of ***potential difference*** (5 pts).

See next page for solution . . .



Consider the meaning of potential difference:

$$\Delta V \equiv V_{\text{(at a place of higher potential)}} - V_{\text{(at a place of lower potential)}} \equiv V_H - V_L$$

$$V_H - V_L \equiv \int_H^L \vec{\mathbf{E}} \cdot \overrightarrow{dr}$$

Each Volt of Potential is a Joule per Coulomb:

The potential difference between two locations is

the work 'per charge' that the field will do

to restore each + 1 Coulumb 'test charge'

to a more stable location

(to bring each test charge closer, that is, back to 'ground').

Field lines, given this definition, always point

from places of Higher Potential to places of Lower Potential:

Locations closer to net positive charge distributions are places of higher potential.

In this case, we will integrate from the 'High Voltage' point at (5, - 6) to the 'Low Voltage' point at (10, - 18).

$$\int_H^L \vec{\mathbf{E}} \cdot \overrightarrow{dr} = \int_{H=P_H(5,-6)}^{L=P_L(10,-18)} \vec{\mathbf{E}} \cdot \overrightarrow{dr}.$$

Note : On Power Supplies, Batteries and other commercial devices, that's what the '+' and '−' mean:
locations of comparatively higher and lower electrostatic potential.

But the limits of integration are icing on the cake;
the integration itself is the more fundamental issue and worth more points in an exam.
Let's look at it first.

$$\Delta V = \int_H^L \vec{\mathbf{E}} \cdot \overrightarrow{dr}$$

What is $\vec{E}$? That was the whole purpose of deploying Gauss's Law:
To find $\vec{E}$ AS A FUNCTION of r.

From II(c)(iv), above, we have:

$E = \dfrac{1}{2\pi\varepsilon_0}\dfrac{\lambda}{r}$. So, here:

$$\Delta V = \int_H^L \vec{\mathbf{E}} \cdot \overrightarrow{dr} = \int_H^L \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}dr$$

$$\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\int_H^L \frac{1}{r}dr \quad . \quad . \quad .$$

$$\Delta V = \left(\frac{\lambda}{2\pi\varepsilon_0}\right)\ln r\Big]_{r_H}^{r_L}.$$

continued on next page . . .

The field function has been integrated.

All constants are known.

(λ has been given as 5 C/m.)

It's now time to evaluate -

so as to find an actual numerical answer (in Volts).

Remember:

$$\Delta V = \int_{r\,at\,P(5,-6)}^{r\,at\,P(10,-18)} \vec{\mathbf{E}} \cdot \overrightarrow{dr}$$

Just like calculating Work,

$\boldsymbol{r}_{\parallel} \equiv$ Displacement (from Charge Line. to Point of Interest), BUT

ONLY the Component PARALLEL to Field Line (direction of force).

How should we think about those limits?

They might well look confusing,

but only because our priorities have shifted a bit since the beginning of the problem.

The points were presented in a manner most consistent

with the rest of the problem:

as coordinate pairs on a (2-D) Cartesian plane.

Now that electric potential has been introduced, however,

we want to picture the locations as endpoints on some (1-D) line-- of integration.

One of many general ways to see/calculate the distance
between a line and a point is below.
You can get this distance whatever way you like;
you can even approximate it without penalty.
The important thing is to recognize that
'distance from a line' necessarily and exclusively
refers to paths perpendicular to that line.
This is mathematically true by definition,
and it is emphatically true in this physics context:
We are summing DOT PRODUCTS between field and the displacement.
The field lines are all perpendicular to the line of charge.

In general, the distance between a line and a point =

$$\frac{ax_0+by_0+c}{\sqrt{a^2+b^2}}.$$

Our charge line is:

$$y=\frac{12}{5}x-12.$$

The expression of our line
most convenient for this context is:

$$\frac{12}{5}x-1y-12=0.$$

$$That\ is,$$

$$12x-5y-60=0.$$

$$So,$$

$$r_H=\frac{12x_0-5y_0-60}{\sqrt{25+144}}=\frac{12x_0-5y_0-60}{13}$$

$$r_H=\frac{12(5)-5(-6)-60}{13}=\frac{\cancel{60}+30-\cancel{60}}{13}$$

$$r_H\approx 2.31\,m\,.$$

continued on next page. . .. . . →

r(L) can be found similarly (from same line with, therefore, same equation):

$$r_L = \frac{12x_0 - 5y_0 - 60}{13}$$
$$r_L = \frac{12(10) - 5(-18) - 60}{13} = \frac{120 + 90 - 60}{13} = \frac{150}{13}$$
$$r_L \approx 11.5$$

SO:

$$\Delta V = \int_{r_H \approx 2.31}^{r_L \approx 11.5} \vec{\mathbf{E}} \cdot \overrightarrow{dr}$$

$$\Delta V = \left(\frac{\lambda}{2\pi\varepsilon_0}\right) \ln r \Big]_H^L$$

$$\Delta V = \left(\frac{\lambda}{2\pi\varepsilon_0}\right) \ln r \Big]_{r_H \approx 2.31}^{r_L \approx 11.5}$$
$$\Delta V = \left(\frac{\lambda}{2\pi\varepsilon_0}\right) [\ln(r_L) - \ln(r_H)]$$

$$here, \lambda = 5\,C/m.$$

$$\Delta V \approx \left(\frac{5}{2\pi\varepsilon_0}\right) \ln r \Big]_{r_H \approx 2.31}^{r_L \approx 11.5}$$

$$\Delta V \approx \left(10 \times \frac{1}{4\pi\varepsilon_0}\right) \ln r \Big]_{r_H \approx 2.31}^{r_L \approx 11.5} \approx (10 \times K_e) \ln r \Big]_{r_H \approx 2.31}^{r_L \approx 11.5}$$
$$\Delta V \approx (1 \times 10^{11}) \ln r \Big]_{r_H \approx 2.31}^{r_L \approx 11.5}$$
$$\Delta V \approx (1 \times 10^{11}) \ln r \Big]_{r_H \approx 2.31}^{r_L \approx 11.5}$$
$$\Delta V \approx (1 \times 10^{11}) [\ln(11.5) - ln(2.31)]$$
$$\Delta V \approx (1 \times 10^{11}) [2.44 - .837]$$

$$\boxed{\Delta V \approx 1.60 \times 10^{11}\, Volts}$$

Note: This amount of 'voltage' is preposterously high.
But so is 5 Coulombs of charge for each of the preposterously large meters we investigate in this problem.
And the number 5 is easier to work with than, say, some other numbers.
As always, we have to make choices:
Chalk them up to 'conservation of convenience.'

Also NOTE: This question would have been slightly easier to picture and approach (though no less challenging to solve) had the second given point looked more like something such as, for example, (12, -5). The second given point was miscalculated, so did not look particularly encouraging.

The problem is the same and solvable by the same method either way. The question, however, is likely to seem clearer and more approachable if the two given points lie along a line manifestly perpendicular to the original line of charge. In such case, the relevant displacement (and therefore path of integration) is directly from one point to the other. Such was the intention of the question, but the second point was miscalculated. As the question is presented, we still solve precisely the same way – 'walking' and integrating along a field line directly away from the charge line, ignoring all other components of displacement, from the first point of interest to the second. When we do not end up 'standing' on the second point of interest, it is undoubtedly harder to see, remember and believe that we need only count the component of displacement which is parallel to the field line. It is nonetheless true either way.

vi. Batteries and power supplies are generally used to supply ***Electric Potential Difference*** ('Voltage') to circuits, thereby allowing us to operate electric and electronic devices. If, according to the question/answer ('v') above, a voltage automatically exists in the space near this charged wire, why do we bother troubling with anything other than this seemingly simple set-up when we wish to operate electrical devices?

Put more specifically:

What about this scenario of One Long Net-Charged Wire does NOT constitute an operable electric circuit?

Describe and discuss the fewest items you would need to add and/or the fewest things you would need to do

in order to turn this situation
(a long straight wire of uniformly dense net positive charge)

into a simple & functional CIRCUIT.

Your response MUST include at least two complete sentences of English
AND at least one clear and specific picture.

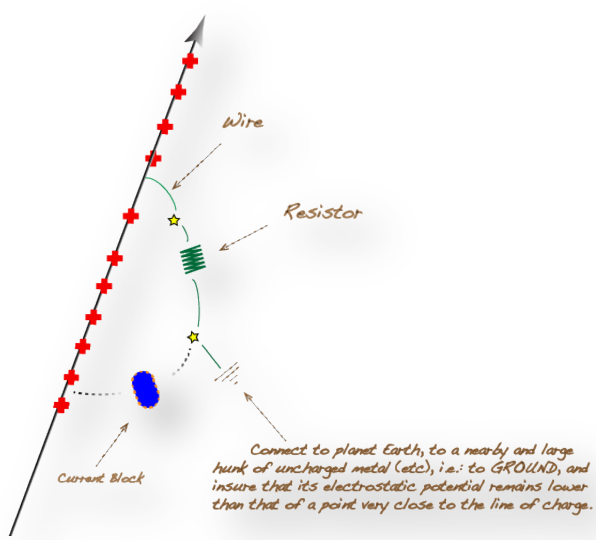
Both the sentences and the picture must be made originally by you (4 pts).

This situation gives charges a reason to accelerate, but it does not provide a (conductive) path through which they (electrons) are free to do so. Furthermore, we have no reason to be confident that the potential difference provided by a line of charge will remain available or at predictable values once charges begin to flow—particularly if the charges come from and thus deplete the line itself. Finally, we have nothing in place to insure or safely harness the conservation of energy: Were charges somehow to begin flowing in a closed loop, the flow needs to be tempered by some significantly non-conductive material which will decrease their electric potential energy – perhaps converting it to a thermal or otherwise useful form. . .

Note: In other words, we do indeed have a potential difference between two spots, but that's more likely to be true than not in any arbitrary region of space and time. What we need – and what human civilization did not begin to develop as a complete package until the nineteenth century – are:

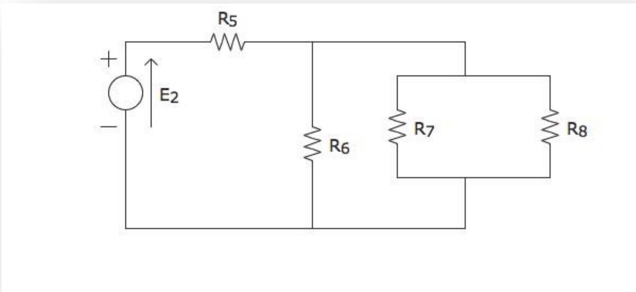
1) a way to maintain the particular charge imbalance underlying a potential difference even as charges themselves begin to move: This is the heart of a *Voltaic Cell* and therefore of a *Battery* (or *Power Supply*);
2) a closed loop made of largely conductive material: *Wire*;
3) at least one finite and deliberately placed region with a known extent of low conductivity: *Resistance*. (The path that allows for flow could be the same as the path that hinders flow (it could simply be somewhere in the middle of the conducting-insulating spectrum of materials), but then it would be very difficult to control, modify or analyze.)

So, . . . Make sure that the length of the line of + charge is indeed way beyond the dimensions of everything else we add: Insure that the λ can function as a 'reservoir' of net charge. Connect a strand of wire from the reservoir to one end of a resistor. Connect the other end of the resistor to r_L or, better, to some equivalently huge yet uncharged and reasonably conductive nearby material – such as planet Earth. Allow the huge and neutral object to act as a reservoir for receiving net +'s without disturbance and rely on it to remain r_L no matter how intricately things are added to the circuit, i.e. to serve as the 'Ground' for all conceivable potential energy comparisons.



III. An actual CIRCUIT (15 pts).

Examine the following circuit (values provided directly to its right).



E2	9 VOLTS
R5	500 OHMS
R6	600 OHMS
R7	700 OHMS
R8	800 OHMS

SHOWING ALL WORK, Determine:

a) The ***current*** flowing through each and every RESISTOR (3 pts (i), 2 pts (ii), 1 pt each of (iii) and (iv)).

i. ***I*** (R5) =

$$\frac{1}{R_{eq(R6,\,R7,\,R8)}} = \frac{1}{600} + \frac{1}{700} + \frac{1}{800}$$

$$\frac{1}{R_{eq(R2,\,R3,\,R4)}} \approx 4.34 \times 10^{-3}\,Ohms^{-1}$$

$$R_{eq(R2,\,R3,\,R4)} \approx 230\,Ohms$$

Therefore,

$$R_{eq(R1,\,R2,\,R3,\,R4)} \approx 500\,Ohms + 230\,Ohms$$

$$R_{eq(R1,\,R2,\,R3,\,R4)} \approx 730\,\Omega$$

$$I_{Battery} \equiv I_{MainLoop} \equiv \frac{\varepsilon}{R_{eq}}$$

Here, $I_5 = I_{Battery}$:

$$I_5 \approx \frac{9\,Volts}{730\,Ohms} \approx 1.23 \times 10^{-2}\,Amperes$$

$$\boxed{I_5 \approx 12.3\,milliAmperes}$$

ii. $\boldsymbol{I}$ (R6) = . . .

Now, between ANY TWO POINTS *in* a circuit,

$$I = \frac{\Delta V}{R}.$$

A central implication of this relation (Ohm's Law) is this:

$$I \propto \frac{1}{R}$$

Given a choice between two current branches,
the portion (fraction) of current that 'chooses' one particular branch is necessarily
INVERSELY proportional to the the fraction of total resistance found in that branch.

Mathematically, treatment of an inverse proportionality might seem 'obvious', but remember:
.... You can rely on the seemingly simple pattern 'one goes up, the other goes down'
IFF the third term of the equation — — the one here referring to potential drop — — is a CONSTANT.
ΔV must be one single value applicable to both branches, NOT, for example,
some kind of 'total value that gets split' between the two!

Indeed. for any two paths or devices in parallel, the potential difference
is necessarily a constant . But this somewhat surprising (hard to remember) idea
comes from physics, not from math:

EVERY CHARGED PARTICLE - no matter which way it goes -
and therefore EVERY INDEPENDENT PATH that any charge might possibly follow
is independently subject to the laws of physics.
EVERY POSSSIBLE PATH must therefore INDEPENDENTLY
OBEY ENERGY CONSERVATION.

So, any and all ways to get from point A to point B
within the same circuit
must do the same work
and impose the same energy effect
on each Coulomb of charge:

Therefore,
The potential drops across any two 'parallel' (independent) routes through a circuit
Are necessarily identical.

It is crucial that you are comfortable with the reasoning behind this certainty — —
More often than not, a person who feels hopelessly stuck
in the middle of a circuit problem is forgetting this vital piece of information.
And more often than not, s/he is forgetting it
because s/he doesn't really believe it. . .

(continued on next page).

So, . . .

$I \propto \frac{1}{R}$

This ~ 12.3 milliAmperes of current splits into three different portions,

but, much like considering forces when applying Newton's 2nd Law,
it is far clearer and more constructive to think of things
'like a charge': ONE CHOICE AT A TIME.
That is, each charge FIRST faces a split between Way #1 (through R6) or the remaining set of Ways: #2&#3, . . .
THEN a split between Way #2 (through R7) or Way #3 (through R8)

In other words, there really is no meaning or utility to any notion of a '3 – Way Choice'.
That's not a choice. That's a mess.

MUCH like this paragraph
might cease to be... if you are willing to slog through a bit more and ....
continue reading on the next page ... .

SO: . . .
R5 (500Ω) is in series with the battery: ALL .12.3 mA come through this resistor.

Then this 'Main Loop' of current (or 'Battery Current')
(two equally acceptable terms for, essentially, the trunk of a tree)
splits into a total of $\boxed{973}$ possible Ohms:
a 600 Ω path vs a 373 Ω path

$$because\left(\frac{1}{700}+\frac{1}{800}\right)^{-1} \approx 373\,.$$

So, $\frac{373}{973}$ of 12.3 mA flows through the the 600Ω ≈ 4.72 mA
and $\frac{600}{973}$ of 12.3 mA flows through the remaining split (700 Ω / 800 Ω) ≈ 7.58mA.

Then, between that 700 Ω and the 800 Ω (a total of 1500 Ω):

$\frac{800}{1500}$ of mA flows through the 700 Ohm ≈ 4.04 mA
and $\frac{700}{1500}$ of 24.1 mA flows through the 800 Ohm ≈ 3.54 mA.

In conclusion:

$$\boxed{\begin{array}{l} \textbf{\textit{i.}}\quad \boldsymbol{R_5}\,\mathbf{(500\,\Omega)}:\quad \boldsymbol{I_5 \approx 12.3\,mA.} \\ \textbf{\textit{ii.}}\quad \boldsymbol{R_6}\,\mathbf{(600\Omega)}:\quad \boldsymbol{I_6 \approx 4.72\,mA.} \\ \textbf{\textit{iii.}}\quad \boldsymbol{R_7}\,\mathbf{(700\Omega)}:\quad \boldsymbol{I_7 \approx 4.04\,mA.} \\ \textbf{\textit{iv.}}\quad \boldsymbol{R_8}\,\mathbf{(800\Omega)}:\quad \boldsymbol{I_8 \approx 3.54\,mA.} \end{array}}$$

b) The ***potential difference*** across each and every RESISTOR (2 pts each).

i. ***ΔV*** (R5) =

$\Delta V = IR.$

So...

$\Delta V_5 = I_5 R_5, etc.$

$$\Delta V_5 = I_5 R_5 \approx (12.3\,mA)(500\,\Omega)\ldots$$

$\boxed{\Delta V_1 \approx 6.15\,Volts.}$

From Energy Conservation,
It SHOULD be the case that:

$$\Delta V_3 \approx \Delta V_2 (\approx \Delta V_2) \approx 2.85\,Volts...$$

Let's Check:

$\Delta V_2 \approx (4.72\,mA)(600\Omega)$

ii. $\boxed{\Delta V_2 \approx 2.83\,Volts \checkmark!}$

Check:

$\Delta V_3 \approx (4.04\,mA)(700\ \Omega)$

iii. $\boxed{\Delta V_3 \approx 2.83 Volts\ \checkmark!}$

And, again, this time from the definition of 'parallel configuration',
It SHOULD be the case that

$$\Delta V_4 \approx \Delta V_3 \approx \Delta V_2 \approx 1.6 mA...$$

Check:

$\Delta V_4 \approx (3.54\,mA)(800\,\Omega)$

iv. $\boxed{\Delta V_4 \approx 2.83\,Volts \checkmark!}$

CHOOSE EITHER OF THE TWO FOLLOWING PROBLEMS:

DO ALL OF *FOUR (IV)*

OR

DO ALL OF *FIVE (V)...*

AFTER THAT, NO CHOICE:

YOU MUST DO
ALL OF
Six (6).

IV. Superposition (20 pts).

Remember Lab #4? Remember how creatively and intensely you endeavored to construct your own wave propagation out of nothing but a bunch of home-grown yet identical oscillators – each moving back and forth with the same angular frequency, but each 'starting' its cycle with a different yet predictable ***phase*** delay (or head-start)? Remember how it was all fuzzily understood or acutely misunderstood mumbo-jumbo at first, but then eventually coalesced into a surprisingly vivid and demonstrative computer-generated animation or flip-book from which, by gum, actual data could actually be inferred and analyzed, by gum? In short, remember how what you eventually put together actually did look pretty neat and might, in fact, almost have been completed into a coherent conclusion regarding the relationship between two differential equations – were somebody to have actually gazed deeply into the beauty of your almost completed product?

Right. I saw a great number of them and they actually did blow me away – with their potential to clarify the nature of waves, just as long as somebody was willing to use them for such a purpose.

SO:

For this question, You MUST submit electronically.

In your blue book (or general answer sheets), when it is the proper place to answer this question, simply write:

"IV. SUPERPOSITION: See electronic submission, entitled ____________________".

Then make use of the "Submit" buttons on the lecture web-page to submit your response (to the questions below). Your response MUST be a file or (preferably) folder/zip-file/etc. of files, named "P204Exam2.71.Lab4.LastName.FirstName".

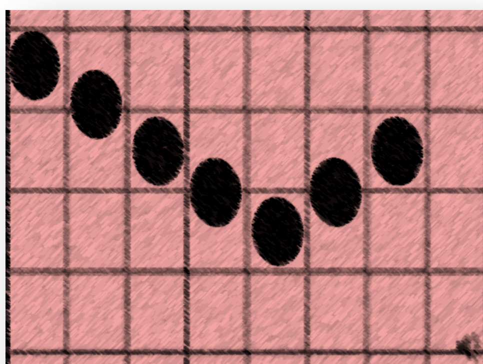
Your response will respond to the following.

A. If the demonstration/model/simulation you created for Lab #4 was electronic, such as a PowerPoint animation, then submit the animation itself – along with all/any instructions necessary for the most appropriate way to run and observe the simulation.

Assuming that your simulation was electronic, then follow the entirety of this step (A) and then skip (B) – proceeding directly to instruction (C), below. If your simulation was ***not*** electronic in nature, then do ***not*** skip (B).

Below is an example of the kind of thing to submit and a perfectly acceptable format in which to submit it;

The substance of this submission is not necessarily perfectly accurate nor necessarily worthy of full credit. Yet it is not necessarily not neither, neither.



B. If the demonstration/model/simulation you created for Lab #4 was not electronic in nature, then capture the best possible video of your demonstration and submit that. With the video, submit any instructions necessary for the most appropriate way to run and observe the simulation.

Below is an example of the kind of thing to submit and a perfectly acceptable format in which to submit it;

The substance of this submission is not necessarily perfectly accurate nor necessarily worthy of full credit. Yet it is not necessarily not neither, neither.

C. Whether you followed instruction (A) or (B), above, please note the following:

NOTE for (A) and (B): You can and should assume that your audience is in possession and serviceable command of commonly known standard applications intended to manage such files (e.g. PowerPoint, Flash, RealPlayer, etc.), but not of applications that are particularly exotic, cutting edge or illegal. If your file requires an application generally familiar only unto a flute-playing puff of aether essence named Uncle Zoltar who hacks enchantments into the corporate headquarters of World of Warcraft, then please come up with a different approach.

Submission according to (A) or (B) is worth 10 pts.

* * *

D. By ANY ORIGINAL MEANS YOU WISH (paragraph, drawing, cartoon, sonnet, blues trilogy, etc.),

provide an original yet PRECISE & THOROUH explanation for the concept (below) that you demonstrated and considered in ***Lab #8***:

Our mathematical expressions for

ACTIONS-AT-A-DISTANCE

that occur between two particles
tend to be
inverse functions of
the square of the distance
between the particles.

WHY?!

The form of your response can be as 'creative' and unexpected as you wish,
but you must FULLY answer the question (10 pts).

V. **B-Fields (20 pts).**

Be brief but complete and precise regarding all of the following:

A. According to Ampere's historic experimental finding, two light, straight, long wires carrying currents going in parallel directions will do WHAT to each other (1 pts)?

B. Ampere's experimental finding is generally explained by a belief that moving charges create WHAT (1 pts)?

C. Draw a magnetic field line diagram for a long straight current:

   a) Head-On: As though the current is coming out of the whiteboard toward your eye (1 pts).

   b) Side-View: As though the current is traveling in a straight line from one side of the whiteboard toward the other (1 pts).

D. Concisely explain the essential differences between the "Dot Product" and the "Cross Product" for the multiplication of two vectors (3 pts).

E. Write down a clear and complete expression (EQUATION!) for the magnetic field as a function of charge, velocity and displacement from the charge (2 pts).

F. Write down clear and complete expression (EQUATION!) for the magnetic field as a function current, length and displacement from the current (1 pts).

G. Write down a clear expression for the magnetic force as a function of current, length, and magnetic field (1 pts).

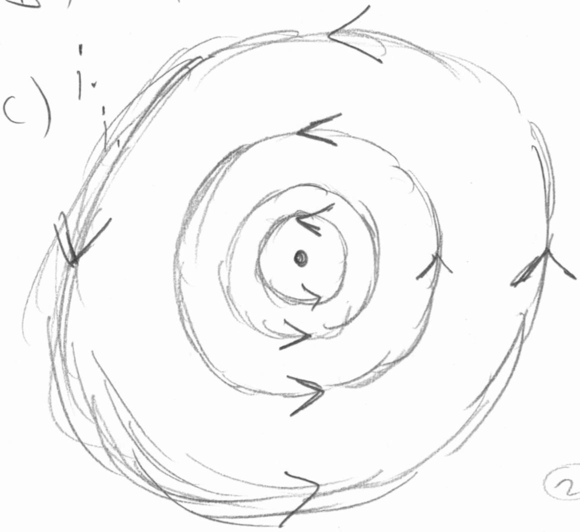
H. Concisely explain the essential differences between the behavior of electric field lines from charges and the behavior of magnetic field lines from charges (3 pts).

B FIELDS

a) ATTRACT!

b) A MAGNETIC FIELD!

c) i.

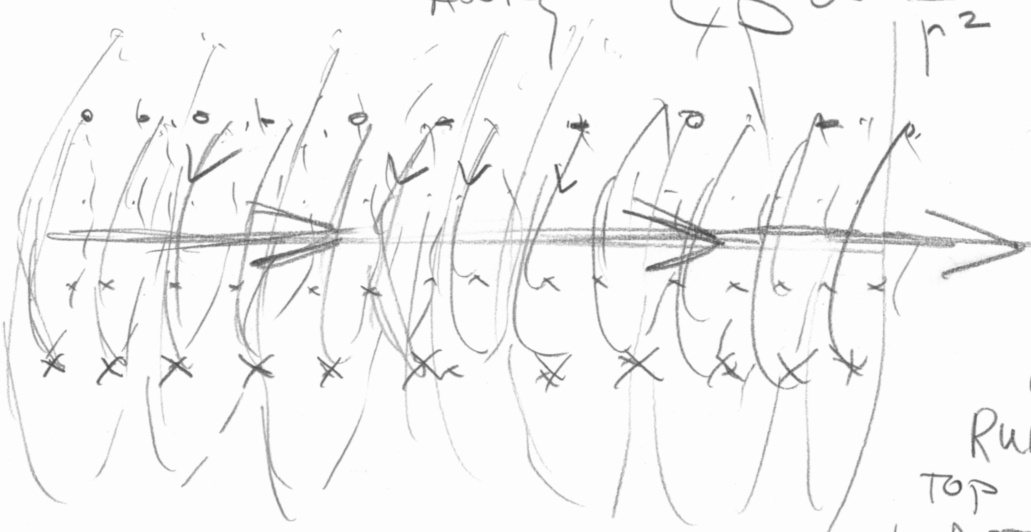


① CONCENTRIC (closed) CIRCLES ENCLOSING THE STRAIGHT CURRENT,

② RUNNING COUNTER-CLOCKWISE (right-hand rule)

③ getting LESS AND LESS DENSE AS THEY get FARTHER AND FARTHER AWAY ($B \propto \frac{1}{r^2}$)

ii.



CONCENTRIC CIRCLES RUNNING FROM TOP out of page to BOTTOM in page

IV. B-FIELDS

d) $$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\, dq\, \frac{\vec{v} \times \hat{r}}{r^2}}$$ ← BIOT-SAVART LAW

e) $$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\, \frac{I\, d\vec{l} \times \hat{r}}{r^2}}$$

(LIKE $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$ FOR ELECTROSTATIC FIELD)

f) $$\boxed{d\vec{F} = I\, d\vec{l} \times \vec{B}}$$ ← LORENTZ FORCE LAW

(FOR CHARGE, IT WOULD BE $\vec{F} = q\vec{v} \times \vec{B}$)

g) $\vec{A} \cdot \vec{B} \Rightarrow$ multiplication of two vectors that produces a SCALAR. in order to do so, it multiplies the parts of the vectors that interact; even when DIRECTION is NOT accounted for: the dot product multiplies the 2 vector components that LIE ALONG ONE AXIS →

IV. g) ASSUME THE FOLLOWING

{ NOTATION }

HERE AND ON EXAM!

i] IF $\vec{A}$ is the vector $\vec{A}$

then $A \equiv \|\vec{A}\|$ is the

pure MAGNITUDE or LENGTH

of $\vec{A}$. (either A or $\|\vec{A}\|$ can be used.)

ii] IF $\vec{A}$ + $\vec{B}$ ARE TWO VECTORS

THEN Θ = THE ACUTE ANGLE between $\vec{A}$ AND $\vec{B}$ WHEN $\vec{A}$ + $\vec{B}$ ARE PLACED TAIL-TO-TAIL

iii] $\odot$ ≡ A VECTOR POINTING "out of the page" TOWARD THE READER

iv] $\otimes$ ≡ A VECTOR POINTING "into the page" AWAY FROM THE READER

v] $\hat{A} \equiv \dfrac{\vec{A}}{\|\vec{A}\|} \equiv \dfrac{\vec{A}}{A} =$ the pure DIRECTION OF $\vec{A}$

THEN...

(NEXT PAGE...)

NOTATION!

THIS PAGE CLARIFIES

IV. B-FIELDS

g) (CONTINUED)

i] $\boxed{\vec{A}\cdot\vec{B} \equiv AB\cos\Theta}$

$$\frac{(\vec{A}\cdot\vec{B})}{\|\vec{A}\cdot\vec{B}\|} \equiv \emptyset,$$ ie.: $\boxed{\vec{A}\cdot\vec{B}}$ HAS NO DIRECTION

BECAUSE THE DOT PRODUCT PRODUCES A PURE SCALAR.

ii] THE CROSS PRODUCT IS THE multiplication of two vectors that produces a vector: it accounts for the DIRECTION of each vector being multiplied and it accounts for them with equal "weight" or "priority" — it therefore multiplies the 2 vector components that lie along DIFFERENT — ie: PERPENDICULAR — AXES AND yields a vector equally $\perp$ TO BOTH ... →

IV B-FIELDS

g) (CONTINUED)

ii] CONTINUED — the X-PRODUCT

$$\|\vec{A} \times \vec{B}\| \equiv AB\sin\Theta$$

$$\frac{\vec{A} \times \vec{B}}{\|\vec{A} \times \vec{B}\|} \Rrightarrow$$ THE "RIGHT-HAND RULE" "(#1)"

PLACE THE FINGERS of your RIGHT HAND ALONG $\vec{A}$ SO THAT THEY ARE CAPABLE of SMOOTHLY "CURLING" (BENDING IN TOWARD PALM).

$\equiv$ YOUR THUMB POINTS ALONG DIRECTION of THE RESULTANT VECTOR, $\vec{A} \times \vec{B}$.

IF/WHEN YOU MASTER THIS, YOU WILL NOTE:

A] $\vec{A} \times \vec{B}$ IS ALWAYS $\perp$ TO THE PLANE CONTAINING $\vec{A}$ AND $\vec{B}$

SO THE R.H.R. REALLY JUST TELLS YOU WHICH WAY IS +(ON) AN AXIS THAT IS ALREADY DETERMINED

B] $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$.

IV B-FIELDS (cont'd)

h) ELECTRIC FIELD LINES ARE PRODUCED BY 0-DIMENSIONAL objects (points) OF PURE MAGNITUDE (SCALARS)

* * *

THE ELECTRIC FIELD IS A VECTOR PRODUCED BY A SCALAR multiplying A VECTOR: THE DIRECTION of THE $\vec{E}$-FIELD IS THEREFORE ENTIRELY DETERMINED BY THE DIRECTION of THE ONE AND ONLY VECTOR FROM WHICH IT IS COMPOSED: THIS VECTOR IS $\hat{r}$: THE VECTOR THAT IS DIRECTED FROM THE point CHARGE TO THE POINT IN SPACE "OF INTEREST" $\rightarrow$

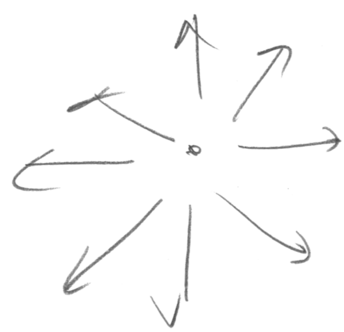
IV

h) CONT'D: $\vec{E}$-FIELDS.

THEREFORE (given page 32)

THE $\vec{E}$-FIELD LINES ARE ALL DIRECTED FROM A POINT CHARGE TO POINTS of INTEREST IN SPACE: THEY RADIATE SYMMETRICALLY FROM A POINT →

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$



IV

h) CONT'D.

THE MAGNETIC FIELD LINES,

ARE PRODUCED BY

1-DIMENSIONAL (LINES) —

WITH DIRECTION,

ie: * * #

THE MAGNETIC FIELD LINE IS

A VECTOR PRODUCED BY

A VECTOR MULTIPLYING ANOTHER

VECTOR.

THE B-FIELD MUST THEREFORE

POINT IN A DIRECTION THAT IS

PERPENDICULAR TO BOTH

THE CURRENT·LENGTH VECTOR

THAT PRODUCES IT AND THE

DISPLACEMENT VECTOR, $\vec{r}$, THAT LEADS TO

A POINT of INTEREST IN SPACE

III h) CONT'D

SINCE EVERY $\vec{B}$-FIELD LINE POINTS PERPENDICULARLY TO BOTH THE CURRENT LENGTH AND THE $\vec{r}$ VECTOR, THEN $\vec{B}$ FIELD LINES MUST FORM CIRCLES AROUND STRAIGHT CURRENTS.

IN GENERAL, $\vec{B}$-FIELD LINES MUST BE CLOSED LOOPS.

I. The POINT.

a) Draw one long and horizontal current-carrying wire a small amount of space above another identical wire – carrying current in the same direction as the wire below it (1 pt).

b) Referring to your picture, you are going to follow a few specific instructions and, ultimately, provide a thorough physics-based explanation of Oersted's historical observation of two current-carrying wires (and what they apparently do when carefully isolated). . .

REQUIREMENTS/DIRECTIONS

i. The explanation you provide MUST include explicit reference to ***EVERY ONE OF*** your answers provided above (A-H). Make a mark (like an asterisk) at the beginning of the sentence each time you have referred to one of the answers (A-H).

ii. The explanation you provide must be strongly focused on the concept of ***DIRECTION***. You need not be too concerned with the concept of magnitude.

iii. Beginning with a little discussion of the field created by the current above (what does it 'look like'? in what directions do the lines point? etc), proceed step-by-step through discussion of the direction of force exerted by this field and, ultimately, how this force affects the current below.

iv. Supplement your initial drawing with any and every visual detail or side-image that might help clarify your explanation.

The goal of all your descriptions is ultimately to make sense out of Oersted's finding in terms of the laws of physics: specifically, why things 'happen' in the direction that they do. Given your picture and the laws of physics, your explanation must ultimately answer this overall question:

Under highly controlled conditions,

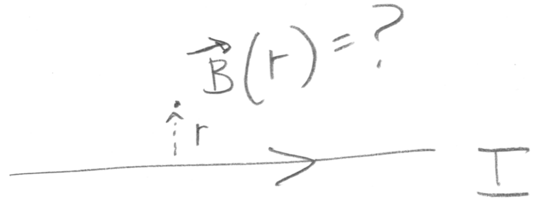
a horizontal current
will be observed to
accelerate up
if a an identically directed current
is located above it

WHY?!

(8 pts).

V.

A)



AS CLOSED "AMPERIAN PATH", CHOOSE A CIRCLE of RADIUS r BECAUSE THE MAGNETIC FIELD LINES ARE CIRCLES

SO A CIRCLE of RADIUS r WILL FIND $\vec{B}$. FIELD of CONSTANT MAGNITUDE AND CONSTANT DIRECTION ALWAYS LYING PERFECTLY PARALLEL TO THE PATH — precisely what the DOT-PRODUCT DEMANDS!

THUS $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{(enc)}$

B/C THE CHOSEN CLOSED PATH IS A CIRCLE $\quad B \oint dl = \mu_0 I_{(enc)}$

$l \equiv$ FULL LENGTH AROUND CLOSED PATH HERE, CIRCUMFERENCE

$B(\;\;) = \mu_0 I_{(enc)}$

$B \cdot 2\pi r = \mu_0$

I A CONTINUED

$$B \oint dl = \mu_0 I_{(enc)}$$

$$B \cdot l = \mu_0 I_{(enc)}$$

$$B \cdot 2\pi r = \mu_0 I_{(enc)}$$

$$B \cdot 2\pi r = \mu_0 I$$

So

$$\boxed{B = \frac{\mu_0}{2\pi} \frac{I}{r}}$$

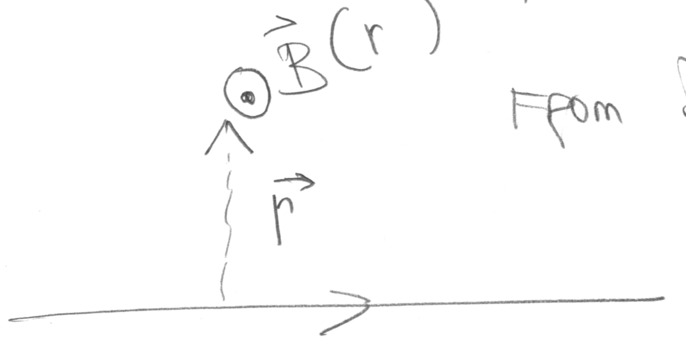
Here, l = CIRCUMF. $= 2\pi r$

$I_{(enc)}$ = ALL CURRENT FLOWING $\perp$ TO AREA BOUND BY CLOSED AMPERIAN PATH

Here, simply I

V

A. (CONTINUED)



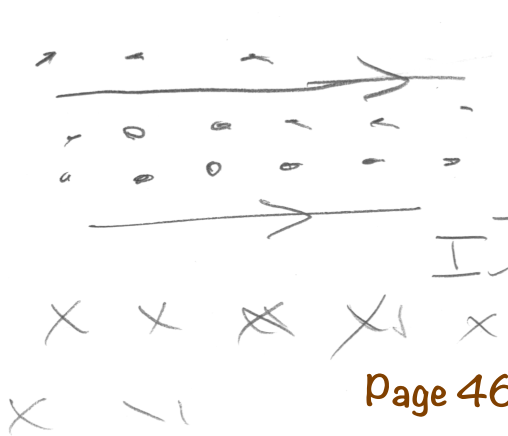
$I\vec{l}_1$

FROM BIOT-SAVART (R.H.R. #1)

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\vec{l} \times \hat{r}}{r^2}$$

THE FIELD AT POINT IN SPACE MUST POINT OUT w/ R.H. THUMB (FINGERS ALONG Il_1 CURL UP TO $\hat{r}$)

BUT NOW IF WE PUT A NEW CURRENT, Il_2 AT THAT OUTWARD POINT FIELD ABOVE Il_1 WE APPLY LORENTZ FORCE!



$I\vec{l}_2$

$I\vec{l}_1$

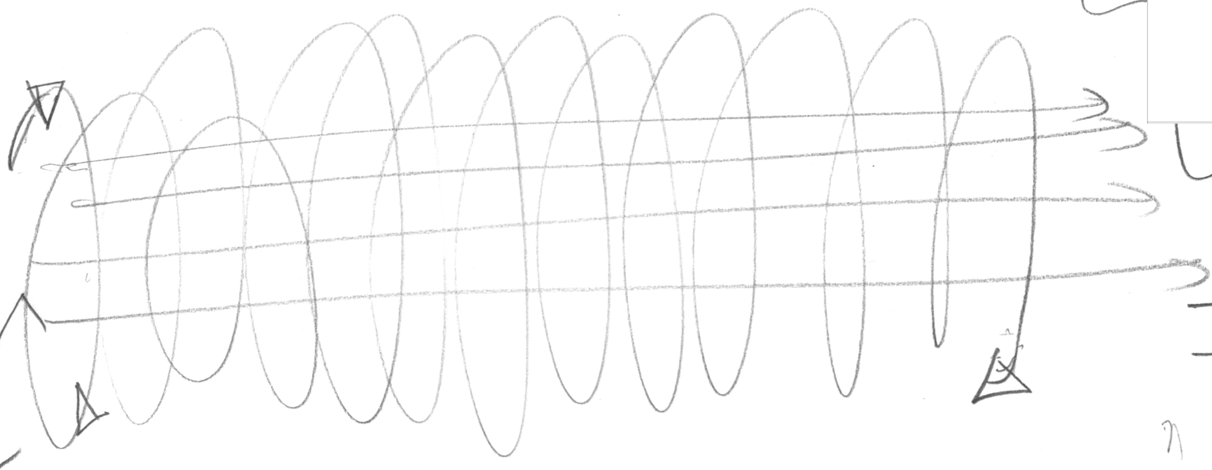
$\vec{F} = I\vec{l} \times \vec{B}$

FINGERS ALONG Il_2 CURL OUT TO FIELD AND THUMB POINTS ↓

THE FORCE ON Il_2 PULLS IT DOWN TO Il_1!

II (B) SOLENOID

n turns per length



I,

DRAW AMPERIAN SQUARE – HALF IN, HALF OUT of SOLENOID,

Let "S" ≡ length of one side

Let "N" ≡ NUMBER of WIRETURNS IN SQUARE

B-FIELD LINES ARE (APPROX) PARALLEL THROUGH AXIS of SOLENOID; ∞ LONG SOLENOID SO THESE PARALLELL LINES MUST go VERY FAR BEFORE CIRCLING right BACK. ∴ DENSITY of LINES OUTSIDE SOLENOID → 0

***

integrate around square counter-clockwise (for CONVENIENCE)

IB (CONT'D)

SOLENOID (CONT'D)

CONSIDER THAT Amperian SQUARE:

THE TWO EDGES THAT ARE

i) $\perp$ TO FIELD LINE CONTRIBUTE 0 TO THE INTEGRAL BECAUSE THE DOT PRODUCT REQUIRES PARALLEL COMPONENTS ($\cos\theta = 0$ here)

ii) the edge OUTSIDE THE SOLENOID CONTRIBUTES 0 because THERE ARE APPROXIMATELY NO FIELD LINES THERE

iii) THE ONE EDGE INSIDE SOLENOID DOES CONTRIBUTE FIELD of CONSTANT MAGNITUDE AND DIRECTION

So $\longrightarrow$

I B SOLENOID.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{(enc)}$$

$$B \oint dl = \mu_0 I_{(enc)}$$

$$B l = \mu_0 I_{(enc)}$$

no r-dependence b/c VERSUS THE MAGNETIC PERI of A CIRC

NOW, IN THIS CASE, $l =$ "S"

AND $I_{(enc)} = N I$ (b/c there are a NUMBER of WIRES FLOWING $\perp$ THROUGH THAT SQUARE'S AREA)

So

$$B l = \mu_0 N I$$

$$B = \mu_0 I \frac{N}{l}$$

↳ we don't know N (depends on our choice of square)

and we don't know l (same dependence)

BUT $n \equiv \frac{N}{l}$ = GIVEN So $\boxed{B = \mu_0 n I}$ CONSTANT.

IC

i) put the edge of one solenoid next to the edge of another and you take parallel wires —
they'll attract! ↑

ii) TURN ONE SOLENOID AROUND AND THE WIRES ARE ANTIPARALLEL
→ REPULSION!

iii) HUNK OF IRON ≤ SOLENOID
→ MADE OF NATURALLY ALIGNED ORBITALS

iv) "NORTH" = DIRECTION of THUMB!

DIRECTION of B FIELD!

VI. *Light: The Excluded Middle (20 pts).*

True/False: Put a "T" in the box next to each claim that appears more true than false.

Put an "F" in the box next to each claim that appears more false than true. (1 pt each).

1. As long as a wave's medium does not change in any respect, then the speed of that wave will not change.	T
	XXXXXX
2. According to Faraday's Law, if the magnetic flux through some open area does not remain constant in time, then an electric potential will be induced in the closed path bounding that area.	T
	XXXXXX
3. ***Nearby but outside*** a charging capacitor ($q > 0$), the magnetic field has a magnitude of 0.	T
	XXXXXX
4. ***Inside*** a charging capacitor ($q > 0$), 0 Coulombs per second of electric charge flows from one plate to the other.	F
	XXXXXX
5. In a circuit containing a capacitor, no current can flow through the wires until the capacitor is fully charged.	F
	XXXXXX
6. Given an open n-dimensional region of space, the region's boundary is a closed region of dimension n-1.	T
	XXXXXX
7. Bounded by any closed surface is a unique volume.	T
	XXXXXX

8. Bounded by any closed path is a unique area.	F
	XXXXXX
9. If an Amperian loop is drawn outside a charging capacitor, then the area bound by the loop must lie outside the capacitor.	F
	XXXXXX
10. Inside a charging capacitor, 0 magnitude of electric flux flows from one plate to the other.	F

	XXXXXX
11. Maxwell's Displacement Current correction was added to Gauss's Law and allowed Gauss's Law to apply to steady currents.	F
12. According to the explanation of Maxwell's Displacement Current, if the magnitude of an electric flux through an open area changes in time, a magnetic field will be induced in the closed path bounding that area.	T
	XXXXXX
13. Mutually induced electric and magnetic fields must be perpendicular to each other.	T
	XXXXXX
14. Given the function $I(t) = \frac{1}{6}at^3$, no matter how many times you differentiate with respect to t $(\frac{dI}{dt}, \frac{d^2I}{dt^2}, \frac{d^3I}{dI^3}, \ldots)$, you will never arrive at the answer "0".	F
	XXXXXX
15. Given the function $I(t) = \frac{1}{6}e^{3t}$, no matter how many times you differentiate with respect to t $(\frac{dI}{dt}, \frac{d^2I}{dt^2}, \frac{d^3I}{dI^3}, \ldots)$, you will never arrive at the answer "0".	T
	XXXXXX
16. If you find charge in some lab, you will necessarily find electric and/or magnetic field in that lab.	T
	XXXXXX
17. If you find electric and/or magnetic field in some lab, you will necessarily find charge in that lab.	F
	XXXXXX
18. When electric and magnetic fields mutually induce each other in a perpetual pattern, that pattern satisfies the wave equation.	T
	XXXXXX
19. $\frac{1}{\sqrt{\varepsilon_0\mu_0}}$ = the speed at which mutually inducing electric and magnetic fields propagate through a vacuum.	T
	XXXXXX
20. $\frac{1}{\sqrt{\varepsilon_0\mu_0}}$ = the speed at which light travels through a vacuum.	T
	XXXXXX
21. *** BONUS *** You will live long and prosper; the net force will be with you.	T