Quiz 1 Name: ____________________________
Physics 204: Yaverbaum June 13, 2011

Quiz:
Simple Harmonic & Wave Motion

I) **DERIVING THE 1-D WAVE EQUATION** (50 pts).

A. **Diagram** (10 pts):
(3 pts for neatness/clarity, 3 pts for completeness, 4 pts for labels).

Draw a neat and clear diagram of an element of a light, tight string, mass ***dm***, linear mass density, $\boldsymbol{\mu}$, length ***l*** (the letter lower case "el", not the numeral "one").

NOTE WELL: Make certain that this string element is being captured in a "snapshot" of some arbitrary orientation: There is no reason that the string should be shaped in a special, stable, uniform or symmetric way.

Indicate and label the presence of an external force ("pull") applied to the string—one which is directed so as to restore the string to equilibrium.

Include the labeled axes of an appropriate coordinate system.

Include any and every other measurement, parameter or condition that might play a role in a derivation of wave relationships.

If you find that including more than one diagram is preferable (so as to present, for example, a magnification of one part of the diagram), then please do so.

B. **Assumptions** (0 pts):

1. Newton's 2nd Law: $\sum \vec{F} = m\vec{a}$,

2. $sin^2\theta + cos^2\theta = 1$
(so all fundamental Pythagorean and trigonometric relations follow),

3. The speed of a wave propagating through a light, tight string of linear mass density $\boldsymbol{\mu}$ and tension $\boldsymbol{\tau}$ is given by $v = \sqrt{\tau/\mu}$ and

4. The external pull applied to the string ultimately causes a ***small-amplitude*** **wave** to propagate along the length of the string.

C. **Derivation** (30 pts)
(15 pts claims [1 -2 pts each], 15 pts justifications [1 – 2 pts each]).

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Given your diagram of the physical situation and given the above assumptions, use the pages that follow to derive an expression for the motion of a wave propagating through a 1-dimensional medium. This expression will be a second order differential equation.

In the left column, write each next claim of the derivation. Next to the claim, in the right column, write a brief reason for that claim. The reason could be expressed in any combination of words, symbols and numbers, but each claim and each reason must be a complete thought.

Make certain to number every claim; that way, subsequent claims can easily refer to prior claims.

If you want to set up your own two-column format (for bigger handwriting or the like), please feel free.

CLAIM	JUSTIFICATION
1.	
2.	
3.	

D. **Meaning** (10 pts)

(3 pts each sentence. 1 pt for each of the following: meaning of symbols, relationship among symbols, relationship between mathematical derivation and physical intuition).

Provide EXACTLY 3 COMPLETE SENTENCES OF ENGLISH explaining the conclusion of the derivation:

For a wave to propagate through a one dimensional medium, what specific changes are taking place at any given points in space and time?

II) **GRAVITATIONAL OSCILLATOR** (50 pts).

A. **Assumptions** (0 pts).

1. Assume that our ***Moon*** is one big solid uniformly dense sphere of mass ***M***, radius ***R***.

2. The force of universal gravitation is given by:

   $F_{gr} = -\frac{GMm}{r^2}$ where $G \approx 6.67\ x\ 10^{-11}\ \frac{Nm^2}{kg^2}$.

3. A comparatively small object of mass m placed in anywhere within a spherical SHELL of substantially larger mass (such as ***M***) will experience a net gravitational force of 0 (because both the gravitational field and the spherically symmetric surface mass thin out as a function of ***r²***).

   (In the expression for universal gravitation, the negative sign is important. It indicates direction and will help toward the end of this problem.)

4. Finally, note that uniform density means that for any two arbitrary volume chunks you select, the mass per unit volume ratios are necessarily the same. That is,

   m/v = M/V.

   In this case, volume refers to the volume of a sphere: $v = \frac{4}{3}\pi r^3$.

B. **Derivation: Force Function** (25 pts)
(12 points claims: 1- 2 pts each; 13 points justifications: 1 – 3 pts each).

Given the above premises, derive an expression for the force of universal gravitation exerted by a large solid sphere on a mass m inside that sphere. Your expression will provide force, $\boldsymbol{F_{gr}}$ as a function of distance, $\boldsymbol{r}$, from the center of the solid sphere.

CLAIM	JUSTIFICATION
1.	
2.	
3.	

C. **Derivation: Diameter Time** (15 pts)
(7 pts claims: 1 – 2 pts each; 8 pts justifications: 1 – 2 pts each).

Using the result you derived in (a), above, imagine that a tunnel is carved out from one place on the Moon's surface to another place on the Moon's surface. Imagine that this tunnel passes through the Moon's center so that it is a diameter. A small mass ***m***, such as a boulder or a subway car, is dropped into one end of this tunnel. The tunnel is empty of anything frictional--including air.

Assume:
1. That the mass of our Moon is approximately 7.36×10^{22} kg,
2. That the radius of the Moon is approximately 1.74×10^{6} m and
3. That ***m*** free-falls through this gravitational tunnel.

DERIVE a NUMERICAL ANSWER to the following question:

How much time will it take for a mass to free-fall through the whole moon?

CLAIM	JUSTIFICATION
1.	
2.	
3.	

c) **Derivation: Chord Time** (10 pts)
(4 pts diagram, 4 pts English, 2 pts mathematical).

Now imagine that a tunnel is carved out from one surface location to another surface location. This time, however, the tunnel is a chord of arbitrary length. It need not pass through the Moon's center.

How much time elapses as a mass travels from one side to the other of an arbitrarily long gravitational tunnel?!

Your answer MUST include:

i. a fully labeled diagram,
ii. at least two complete sentences of English,
iii. at least one complete mathematical/numerical statement (such as an equation).