PRACtiCE MidTerm #1)

Oscillation, Interaction

& Superposition

PHYSICS 204, MARTENS YAVERBAUM, ET AL.

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Name: ____

Section: ____________

SCORE: _______________

THE FOLLOWING RELATIONS UNDERLIE THE MATERIAL:

1) $\sum \vec{F} = m\vec{a}.$

2) $F = -Kx.$

3) $x = Acos(\omega t + \phi).$

4) $\omega \equiv 2\pi f.$

5) $f \equiv \frac{1}{T}.$

6) $\theta \equiv \frac{x}{r}.$

7) $\lim_{\theta \to 0} \frac{sin\theta}{\theta} = 1.$

8) $\frac{\partial^2 y}{\partial t^2} = \left(\frac{T}{\mu}\right)\frac{\partial^2 y}{\partial x^2}.$

10) $y = Acos(\omega t - kx).$

11) $k \equiv \frac{2\pi}{\lambda}.$

12) $v = \frac{\omega}{k}.$

13) $\vec{F} = -\frac{GMm}{r^2}\hat{r}.$

14) $G \approx 6.67\ x\ 10^{-11}\ \frac{Nm^2}{kg^2}$

15) $\mu \equiv \lambda \equiv \frac{dm}{dx}.$

16) $\sigma \equiv \frac{dm}{dA}.$

17) $\rho \equiv \frac{dm}{dV}.$

SCORE: ________________

1) UNIVERSAL GRAVITATION FROM *POINT MASSES*.

Two ***point-masses*** of differing magnitudes sit somewhere in a parallel universe -- a universe identical to ours in most respects.

As one might guess, ***you*** generally live somewhat near the center of this universe, and you always have a mass of precisely 1 kg. Today, today things are a bit off-center. Other than you and the two points, little else seems to be happening in the universe.

The two point masses are as follows:

Name	Mass	x-Coordinate	y-Coordinate	Ordered Pair
M_1	8 kg	+5 meters	+15 meters	(+5,+15)
M_2	72 kg	-25 meters	+15 meters	(-25,+15)

YOUR LOCATION: (0,0)

$G \approx 6.67 \times 10^{-11} \frac{Nm^2}{C^2}$, but you are welcome to leave it as '***G***' as long as you like.

* * *

a) Draw a neat and clear sketch of the situation, as you understand it. Your sketch must express a clear decision as to which directions are designated by + and – on each axis (3 pts).

b) COMPUTE THE NET GRAVITATIONAL FORCE ON YOU -- from the two point masses.

THAT is:

i. In Newtons, determine the **magnitude** of Force you feel at this location of interest (5 pts).

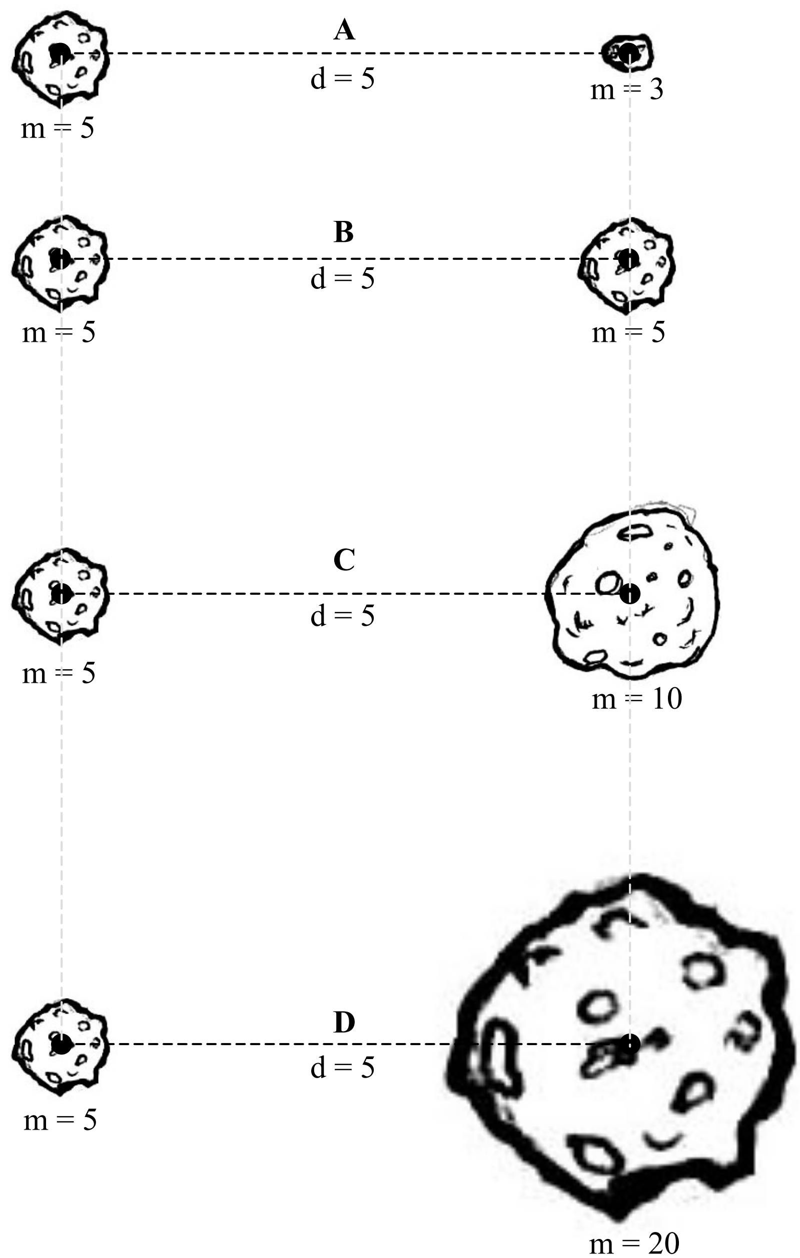
ii. Using degrees **and** points of the compass (such as "20 degrees South-West"), express the **direction** in which you are pulled (3 pts).

c) For the VERY FIRST INSTANT these masses begin pulling on you, what is the magnitude and direction of your instantaneous acceleration?

SCORE: _______________

2. Warm Up: UNIVERSAL GRAVITATION FROM LARGE YET DISTINCT CONTINUOUS MASSES (25 PTS)

Description: The figures below (A – D) each show two rocky asteroids with masses (m), expressed in arbitrary units, separated by a distance (d), also expressed in arbitrary units.



A. **Ranking Instructions:** Rank (from greatest to least) the strength of the gravitational force exerted on the asteroid located on the left side of each pair.

Ranking Order: Greatest 1 ____ 2 _____ 3 _____ 4 ____ Least

Or, the strength of the gravitational force exerted in each case is the same. _________ (indicate with a check mark)

Carefully explain your reasoning for ranking this way:

____
____
____
____

B. **Ranking Instructions:** Using Newton's Second Law, rank the acceleration (from greatest to least) that the asteroids located on the left side of each pair would experience due to the gravitational force exerted on it.

Ranking Order: Greatest 1 ______ 2 _______ 3 _______ 4 ______ Least

Or, the accelerations for each asteroid is the same. _________ (indicate with a check mark)

Carefully explain your reasoning for ranking this way:

____
____
____
____

3) UNIVERSAL GRAVITATION FROM *CONTINUOUS MASSES (25 PTS)* .

The following statements immediately are considered (in this context) ***GIVEN.***

You may assume and rely on them for the problem/proof to follow a bit further down.

Note: In some cases, 'GIVEN' might mean 'self-evident' or 'obvious', but in other cases, it might not. GIVEN might not mean 'obvious'; it can simply mean 'somehow established prior to this discussion'.

GIVEN (for this context) →

1) Planet Earth can be treated as one large SOLID SPHERE of UNIFORM (constant) DENSITY, with constant ('known') mass ***M***, constant ('known') radius ***R***.

2) Given ANY TWO POINT MASSES, say m_1 and m_2, then m_1 will necessarily exert a gravitational force onto m_2 and thereby pull m_2 directly toward m_1.

Hence, the gravitational pull exerted by m_1 onto m_2 is given by force given by:

$$\vec{\mathbf{F}} = -\frac{Gm_1m_2}{r^2}\hat{r}$$

3) The ratio of mass to volume is a constant:

That for any given segment of (3-D) space, ***V***, we expect to find an approximately unwavering amount of mass ***M.***

So, in general, it is fair and useful to say:

$$\frac{m_1}{V_1} = \frac{M_2}{V_2}$$

By VOLUME, here, we mean the volume of a sphere:

$$V = \frac{4}{3}r^3$$

a. SO, given all the above,

DERIVE: F_{gr} = ***[what function of](r)*** **?** (7 pts).

Let F_{gr} stands for the net gravitational force (**dependent variable**) which all the bits of ***M*** together exert on some particle of mass, ***m*** ,

as determined by ***r*** **(independent variable)**;

r stands for the displacement from the center of a solid sphere -- assuming that ***r*** =< ***R*** , that is:

assuming that the particle is located somewhere within the solid sphere.

b. Using the result you derived in (a), above,
imagine that a tunnel is carved out from one place on Earth's surface to another place on Earth's surface.
Imagine that this tunnel passes through Earth's center so that it is a diameter.
A small mass ***m***, such as a boulder or a subway car, is dropped into one end of this tunnel.
The tunnel is empty of anything frictional—including air.
Assuming that ***m*** free-falls through this gravitational tunnel, the key question becomes:

How much time will it take for the mass to reach the other side?

i. Hint: This key question, above, is where all your work for (a) becomes worth it. This is where your understanding of simple harmonic motion becomes relevant. This is where you see why simple harmonic motion is such a sweet concept.

First, find this time as a general expression: as a function of the given and fundamental constants (***G***, ***M***, ***R***) (8 pts).

ii. Second, evaluate your function in order to get an actual numerical measurement — an actual number of MINUTES, in this case, as an answer for time across (5 pts).

In order to obtain a numerical answer for time, use the standard numerical values for Earth's characteristics:

c. Now imagine that a tunnel is carved out from one surface location to another surface location.
This time, however, the tunnel is a chord of arbitrary length. It need not pass through Earth's center.
How much time elapses as a mass travels from one side to the other of an arbitrarily long gravitational tunnel?! (5 pts).

Hint: Again, think hard about what simple harmonic oscillation really means.

Again, this is worth it. Again, it's not as difficult to solve as it may sound,

4) MATH METHODS

a.

$$x = e^{-wt}$$

$$\frac{dx}{dt} = ?$$

$$\frac{d^2x}{dt^2} = ?$$

SCORE: ________________

b. Differential Equations.

i. First, Just Assume that the following function is true and somehow useful.

$$x = A\cos(\omega t)$$

Then find:

$$\frac{dx}{dt} = ?$$

$$\frac{d^2x}{dt^2} = ?$$

ii. **Now switch gears:**

$is\ x = A\cos(\omega t)$ **'a solution'** *to*

$$\frac{d^2x}{dt^2} = -(\omega^2)x?$$

Why or why not?

SCORE: ________________

iii. Now assume: $x = Acos(5t)$

Is x = A cos (5t) 'a solution' to

$\frac{d^2x}{dt^2}$ = -3x ?

Could there be conditions under which $x = A\cos(\omega t)$ *is* 'a solution' *to*

$$\frac{d^2x}{dt^2} = -\left(\frac{K}{m}\right)x?$$

What would these conditions be?

Is $x = e^{-wt}$ a 'solution' to $\frac{d^2x}{dt^2} = -(\omega^2)x$?

Why or why not?

Assume that $i \equiv \sqrt{-1}$.

and Let $x = e^{-i\omega t}$.

$$\frac{dx}{dt} = ?$$

$$\frac{d^2x}{dt^2} = ?$$

Is $x = e^{-i\omega t}$ a 'solution' to $\frac{d^2x}{dt^2} = -(\omega^2)x$?

Why or why not?

SCORE: ________________

5) SHO

A. **Spring**

Consider two possible descriptions for the behavior of some system, below:

i)

$$\frac{\partial^2 J^3}{\partial t^2} = -(LC)J^4;$$
$$LC = 3.14 \times 10^{-5}\sec^{-2};$$
$$J^5{}_0 = 300\, milli - Amperes^4.$$

and

ii)

$$\frac{\partial^2 p}{\partial t^2} = -(\beta^4)p;$$
$$\beta^4 = 3.14 \times 10^5 rad^2/\sec^2;$$
$$p_0 = 300\, Pascals$$

a. Assuming that in each one, ***t*** stands for time
(as measured from some initial moment called 0),

indicate WHICH ONE of the above descriptions could refer to a system behaving in Simple Harmonic Oscillation; simply reply **i** or **ii** (3 pts).

SCORE: ________________

b. For whichever expression you DID CHOOSE to be an SHO, find its ***period*** of oscillation; provide a *number*, measured in seconds (5 pts).

c. For whichever expression you DID CHOOSE to be an SHO, determine the value of the dependent variable at t = 100 seconds from the beginning of measurement. In your final answer, include whatever units are appropriate – according to the facts of that problem (5 pts).

B. **Fall.** (35 pts).

a. (includes all four parts below).

Consider the following differential equation:

$$\frac{d^2r}{dt^2} = -\left(\frac{Qq}{4\pi\varepsilon_0 mR^3}\right) r,$$

in which $\boldsymbol{Q, q, R, m}$ and $\boldsymbol{\varepsilon_0}$ are all non-zero constants,
$\boldsymbol{r}$ is a variable measured in meters
and $\boldsymbol{t}$ is a variable measured in seconds.

i. Show that $\boldsymbol{r = \frac{1}{2}at^2 + v_0 t}$ (in which $\boldsymbol{a}$ and $\boldsymbol{v_0}$ are constants; $\boldsymbol{a \neq 0 \neq v_0}$) ***IS NOT a solution*** to the differential equation presented above.

Your response must be largely *mathematical* yet include at least one clear thought expressed in *English* (7 pts).

SCORE: ________________

ii. Show that $\boldsymbol{r} = \boldsymbol{r_0}\, cos(\omega t + \kappa\chi)$ (in which $\boldsymbol{\omega}, \boldsymbol{\kappa}, \boldsymbol{r_0}$ ***and*** $\boldsymbol{\chi}$ are <u>constants</u>) ***IS a solution*** to the *differential equation* above (4 pts).

iii. according to this solution, how is $\boldsymbol{\omega}$ *related to* the *constant* term(s) in the DIFFERENTIAL EQUATION (4 pts)?

a. according to this solution, in what units must the constant product $\boldsymbol{\kappa\chi}$ be measured (3 pts)?

Assume $\boldsymbol{\kappa\chi = 0, r_0 = 1}$ and $\frac{Qq}{4\pi\varepsilon_0 mR^3} \approx 9 \times 10^9$ s^{-2}
.

b. In seconds, what is the ***period of oscillation*** for the particle described by the above differential equation (7 pts)?

c. find the approximate value for ***r*** when $\boldsymbol{t = 2}$ ***SECONDS*** (7 pts)? CAREFUL.

d. If the value of the constant $\boldsymbol{\varepsilon_0}$ were to triple (while all else remained fixed), then ***what would happen*** to the ***number of oscillations*** we would expect to count ***per hour***? CAREFUL.
(Would the number increase or decrease; by **precisely how much**?) (3 pts.)

SCORE: ________________

b. (includes all four parts below) (35 pts).

Near the surface of Earth, a particle of mass $\boldsymbol{m}$ is dangled from a long string, length $\boldsymbol{L}$; the particle oscillates along a small arc according to the differential equation

$$\frac{\partial^2 \boldsymbol{\theta}}{\partial t^2} = -(\mathbf{20})\boldsymbol{\theta},$$

where $\boldsymbol{\theta}$ refers to an angular displacement measured from the vertical and $\boldsymbol{t}$ refers to time.

The particle's mass is given by $\boldsymbol{m = 8\ kg}$.

The length of the string, L, is constant and was accidentally not recorded by the researchers – but can be deduced from all the other given information.

Whenever the particle arrives at a location of $\boldsymbol{\theta = .2\ radians}$ from the vertical, the particle has no instantaneous speed. On both sides of the vertical, that is, $\boldsymbol{\theta = .2\ radians}$ is repeatedly observed to be a 'turning point' for the particle's periodic motion.

i. Draw a clear diagram of this particle at some arbitrary point during oscillation, making sure to label variables and constants described above (4 pts).

ii. In what ***units*** should the constant (20) be measured (3 pts)?

iii. Approximating to three significant digits,
what is the ***angular frequency*** of this oscillator on a string (4 pts)?

iv. ***What is the particle's approximate SPEED at t = 3 seconds*** (4 pts)?

v. Assume that an experimenter begins measuring time at the instant the particle reaches .2 radians from the vertical. Assume, further, that the only force doing work on this dangling particle is gravity.

(a) ***How much Potential Energy*** does the particle have at ***t = 0 – immediately at the start*** (10 pts)?

(b) Assume that all the above values remain precisely the same, EXCEPT that the ***turning*** points at +/- .2 radians now occur at +/- .6 radians: In other words, the amplitude has been TRIPLED.

Given this particular change, describe what will happen to the value of the ***period*** of time for one cycle (10 pts).

SCORE: ________________

6) THE PUDDING

1) FIRST, PROVIDE A CLEARLY LABELED DIAGRAM which presents the situation, clarifies many of the situational 'givens', and defines constants and variables to be mentioned in your derivation.

2) THEN with fundamental principles of physics, geometry, calculus and English,

DERIVE THE FOLLOWING STATEMENT you choose. That is, show how the statement follows from a series of steps – each of which can be justified as a small and reasonable inference from some step before.

To prove: **"The motion of a small-angle, planar pendulum is approximately Simple Harmonic".**

For this derivation, You May ASSUME:

1) A particle of m is suspended at the end of a long, light string, length L;

2) It is held still at a small angular displacement from an equilibrium position – found at the lowest possible point on the vertical;

3) the small angular displacement is called θ.

4) It is then released – with no starting speed at all – from the small angular displacement, called θ.

5) You may also assume:
a) Newton's 2nd Law,
b) The Small-Angle (Radian) Approximation for the Sine of an Angle,
c) Anything we have already established about the solution to the 2nd order Dif. Eq. for SHO.

(So please first write all the above into your sheet as "Givens"…)

SCORE: ______________