MIDTERM 1: PRACTICE!

PHYSICS 204, MARTENS YAVERBAUM
JOHN JAY COLLEGE, FALL 2017,

POSTED:
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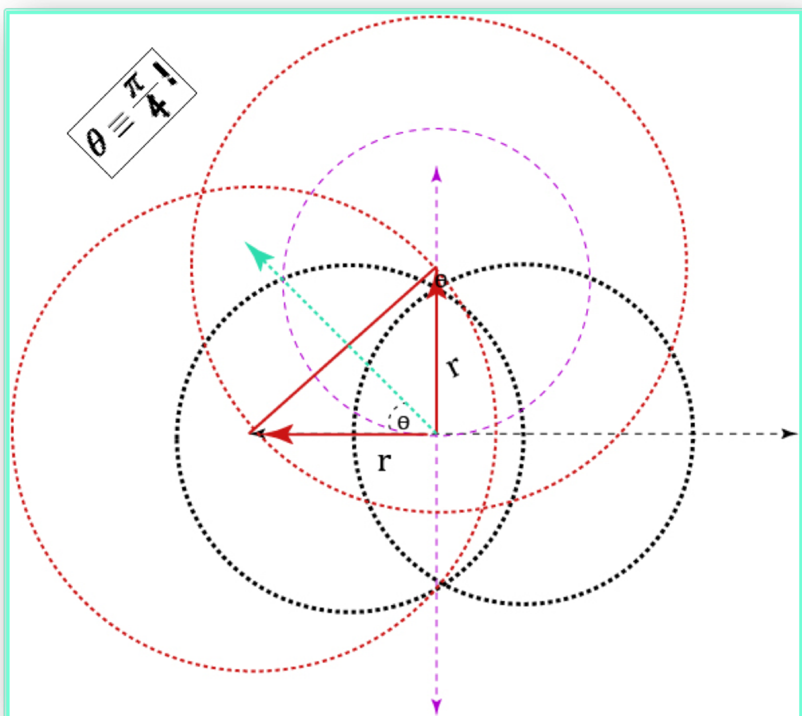

Name: ______________________________

THE FOLLOWING RELATIONS UNDERLIE THE MATERIAL:

1) $\sum \vec{F} = m\vec{a}.$

2) $F = -Kx.$

3) $x = Acos(\omega t + \phi).$

4) $\omega \equiv 2\pi f.$

5) $f \equiv \frac{1}{T}.$

6) $\theta \equiv \frac{x}{r}.$

7) $\lim_{\theta \to 0} \frac{sin\theta}{\theta} = 1.$

8) $y = Acos\ (\omega t - kx).$

9) $\vec{F} = -\frac{GMm}{r^2}\hat{r}.$

10) $G \approx 6.67\ x\ 10^{-11}\ \frac{Nm^2}{kg^2}$

11) $\mu \equiv \lambda \equiv \frac{dm}{dx}.$

12) $\sigma \equiv \frac{dm}{dA}.$

13) $\rho \equiv \frac{dm}{dV}.$

1) MATH METHODS (25 PTS)

Note: $i \equiv \sqrt{-1}$

a) $x_a = -ie^{-i\omega t}$; ω, e and i are all constants

.

i. $\frac{dx}{dt} = ?$ (3 pts)

ii. $\frac{d^2x}{dt^2} = ?$ (2 pts)

b) $z = z_0{}^2 \sin(\omega t + i^3)$; ω, i and z_0 are all constants (5 pts)

i. $\frac{dz}{dt} = ?$ (3 pts)

ii. $\frac{d^2z}{dt^2} = ?$ (2 pts)

c) is $x = -ie^{-i\omega t}$ (see (a), above)

a solution to $\frac{d^2x}{dt^2} = -\omega^2 x$?

Show why yes or no (5 pts).

d) is $z = z_0{}^2 \sin(\omega t + i^3)$ (see (b), above)

a solution to $\frac{d^2z}{dt^2} = -\omega^2 x$?

Show why yes or no (5 pts)

e) Find and verify a $y = f(t)$ which solves a Dif. Eq. described by two facts below:

Your answer will have a ***u*** and a ***t***,

and everything else will be a number $--$ a number which you have figured out!

(***T***, which stands for period, should NOT be in your answer.)

Fact 1:

$\frac{d^2 u}{dt^2} = -\pi^4 u,$

Fact 2:

when $t = 0$, then it just so happens that $u = T$.

(5 pts).

<u>2 Deduce</u>: 30 pts

1) FIRST, PROVIDE A CLEARLY LABELED DIAGRAM which presents the situation, clarifies many of the situational 'givens', and defines constants and variables to be mentioned in your derivation (5 pts).

2) THEN with fundamental principles of physics, geometry, calculus and English,

DERIVE THE FOLLOWING STATEMENT. That is, show how the statement follows from a series of steps – each of which can be justified as a small and reasonable inference from some step before.

To prove: **"The motion of a small-angle, planar pendulum is approximately Simple Harmonic" (25 pts).**

For this derivation, You May ASSUME:

1) A particle of m is suspended at the end of a long, light string, length L;

2) It is held still at a small angular displacement from an equilibrium position –
found at the lowest possible point on the vertical;

3) the small angular displacement is called θ.

4) It is then released – with no starting speed at all – from the small angular displacement, called θ.

5) You may also assume:
a) Newton's 2nd Law,
b) The Small-Angle (Radian) Approximation for the Sine of an Angle,
c) Anything we have already established about the solution to the 2nd order Dif. Eq. for SHO.

(So please first write all the above into your sheet as "Givens"…)