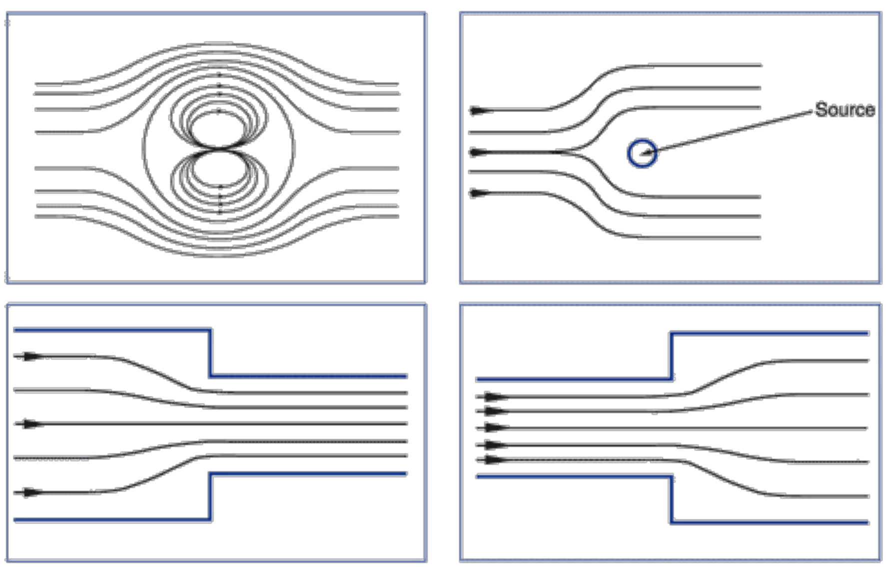
MidTerm #1:

Oscillation, Propagation. . . &

. . . FLOW

(of, for example, a fluid...
such as the occasionally white waters of a river like Hudson...
on, say, April 19, 2015.



PHYSICS 204, WITH HONORS, PROF. D.A. MARTENS YAVERBAUM

JOHN JAY COLLEGE OF CRIMINAL JUSTICE, THE CUNY

FRIDAY, APRIL 15, 2016

Name: ____

Section: ____________

SCORE: ______________

THE FOLLOWING RELATIONS UNDERLIE THE MATERIAL:

1) $\sum \vec{F} = m\vec{a}.$

2) $F = -Kx.$

3) $x = Acos(\omega t + \phi).$

4) $\omega \equiv 2\pi f.$

5) $f \equiv \frac{1}{T}.$

6) $\theta \equiv \frac{x}{r}.$

7) $\lim_{\theta \to 0} \frac{sin\theta}{\theta} = 1.$

8) $\frac{\partial^2 y}{\partial t^2} = \left(\frac{T}{\mu}\right) \frac{\partial^2 y}{\partial x^2}.$

10) $y = Acos(\omega t - kx).$

11) $k \equiv \frac{2\pi}{\lambda}.$

12) $v = \frac{\omega}{k}.$

13) $\vec{F} = -\frac{GMm}{r^2}\hat{r}.$

14) $G \approx 6.67 \ x \ 10^{-11} \frac{Nm^2}{kg^2}$

15) $\mu \equiv \lambda \equiv \frac{dm}{dx}.$

16) $\sigma \equiv \frac{dm}{dA}.$

17) $\rho \equiv \frac{dm}{dV}.$

18) $\frac{1}{2} \rho v^2 + \rho g h + p_1 = \left(\frac{1}{2} \rho v^2 + \rho g h + p2\right) \backslash \rho v^2 + \rho g h + p$

1

SCORE: ________________

Part I: **Exercises** (25 pts)

CHOOSE EITHER (A) or (B). Do ALL of ONE.

A. **Fall** (15 pts).

Consider the following data for planet Earth:

$$\vec{F} = -\frac{GMm}{r^2}\hat{r}; G \approx 6.67 \times 10^{-11}\frac{Nm^2}{kg^2}$$

$$R_E \approx 6.40 \times 10^6\ meters; M_E \approx 6.00 \times 10^{24} kg$$

(Now note the corresponding data for the star called ***Sirius***, the fourth brightest object in Earth's night sky:)

$$R_S \approx 1.22 \times 10^9\ meters; M_S \approx 4.02 \times 10^{30} kg$$

For the remainder of this problem – ESPECIALLY THE LAST COUPLE OF PARTS –

Please assume that the ***DENSITY*** of Sirius is essentially ***CONSTANT***.

(This concept can easily apply to substances that are not solid, for example to liquids.)

a. In meters/sec^2, to three significant digits, compute the approximate free-fall acceleration ($\boldsymbol{g}_i$) of any mass dropped very near Sirius's surface (3 pts).

b. Experts from the future have sent us the following report:
When you – yes, *you* – stand on the surface of Sirius, you are evidently measured to have a ***weight*** of approximately ***12600 Newtons***.

   i. Given the facts above,
   In kilograms, determine your approximate ***mass*** (2 pts).

SCORE: ________________

ii. Imagine:

You somehow find yourself at *precisely the dead center* of this uniform sphere called Sirius (and imagine that you somehow become very very short—like, barely taller than a Euclidean/infinitesimal point).

Under these conditions,

In kilograms, determine your approximate ***mass*** (2 pts).

iii. Imagine precisely the conditions described above. Again, you are at the exact center of Sirius. And you're strangely short.

Under these conditions,

In Newtons, determine your approximate ***weight*** (2 pts).

iv. Some time later, you are located precisely

one fifth (1/5) of the distance from the center to the surface of Sirius.

Under these conditions,

In Newtons, determine your approximate ***weight*** (3 pts).

c. Now assume that one sad day, our sun somehow ceases to be our sun. The mass and radius of the star Sirius are of similar magnitudes to those of our Sun, so, in this hypothetical scenario, we (planet Earth) get pulled into orbit around Sirius.

We are continually pulled toward Sirius, in this scenario, and there are no other significant forces acting on us.

Assume that we (Earth) orbit Sirius at a distance from its surface which is precisely four times as large as the radius of Sirius. (Hint: This means that our total distance from the CENTER of Sirius is . . . ______ many times greater than the radius of Sirius….)

Determine the magnitude of Earth's acceleration while it remains in gravitational orbit around Sirius. Remember: NO OTHER FORCES at play (3 pts).

SCORE: ______________

B. **Spring** (15 pts).

Consider two possible descriptions for the behavior of some system, below:

i)

$$\frac{\partial^2 J^3}{\partial t^2} = -(LC)J^4;$$
$$LC = 3.14 \times 10^{-5}\sec^{-2};$$
$$J^5{}_0 = 300\, milli - Amperes^4.$$

and

ii)

$$\frac{\partial^2 p}{\partial t^2} = -(\gamma^4)p;$$
$$\gamma^4 = 3.14 \times 10^5 rad^2/\sec^2;$$
$$p_0 \equiv p_{(t=0)} = 300\; Pascals$$

a. Assuming that in each one, ***t*** stands for time (as measured from some initial moment called 0),

indicate WHICH ONE of the above descriptions could refer to a system behaving in Simple Harmonic Oscillation; simply reply **i** or **ii** (3 pts).

b. To the one you decided was NOT an SHO, make the smallest possible correction you can—so as to turn it into a proper SHO. (Then leave it alon. And answer all remaining questions with reference to the one you decided was SHO) (2 pts).

c. For whichever expression you DID CHOOSE to be an SHO, find its ***period*** of oscillation; provide a *number*, measured in seconds (5 pts).

d. For whichever expression you DID CHOOSE to be an SHO, determine the value of the dependent variable at t = 100 seconds from the beginning of measurement. In your final answer, include whatever units are appropriate – according to the facts of that problem (5 pts).

SCORE: _______________

NOW, make a fresh choice: CHOOSE EITHER (C) or (D):

C. **Wiggle** (10 pts).

Consider an extremely long and negligibly thick column of fluid:
Consider, for example, air.

Imagine that one end of the air column is subjected to a series of quick but patterned disturbances which result in some sort of patterned tremors or vibrations:
Consider, for example, breathing into a flute.

In your mind's ear, imagine that sound which is soon to emerge from the other end of the air column. There is little to no motivation for us to respond to this comfortable event with a fresh story about some spontaneously generated signal who just popped up to say hello one fine day in A sharp minor. Rather, we roll with our typical and intuitive picture ij which sound has traveled from one end to the other.

But occasionally eventually we pause to ponder precisely how, which or ***what*** in the world could be making the motion when such motion was made. ***Which*** tiny bits of material to make it from one end to the other in order for such an experience to … be. Other than air, what material material could possibly be considered? Not even the remotest hint of the slightest involvement of any other material seems apparent. And when or on what basis did we start referring to AIR as material? By ***material***, we mean solid, no?

No. ***Material*** refers to matter. And *solid* refers to a mere ***one*** out of three distinct states or structural types in which matter customarily finds itself: the arrangement, that is, for which concerns of basic shape identification, maintenance and border patrol are left to the shape itself. Abandon all hope, ye who crave containment. Prepare to climb some walls.

D. Observations strongly suggest that sound travels from one end of the column to the other and strongly discourage any other way of referring to or discussing these observations. No observation shows how to identify, limit, enclose or distinguish that which is a sound from a that which is not a sound. If bits of melody or speech move through air, then it is air molecules – and air only -- which have done the moving. But many a musician has made much on the mildest of un-windy days. (Sound can get to you, that is, without a volume of air flowing toward you in any recognizable fashion.)

You thus realize the following :

When sound travels, it ***propagates*** – according to all the physical trends and mathematical functions customarily associated with wave motion. We can track wiggle values as they cycle repeatedly through a known amplitude. BUT, unlike our string protype, this propagation seems to require no more than one spatial dimension. How so?

SCORE: _______________

The individual oscillations take place on the same axis as the collective propagation: The dependent variable does not measure position along some transverse axis; rather it measures pressure pockets and/or clumps of tghtly clustered molecules...

We thus have,… A typical rhythm of pressure beats, where $\boldsymbol{t}$, $\boldsymbol{x}$ (independent) and $\boldsymbol{p}$ (dependent) are variables and everything else is a constant.

$$p = p_0 \cos(\omega t - kx)$$

1. If compressions really work the same fashion as visible stretches along a string, then you should be able to:

   Show that the above equation IS A solution to the second order differential equation

$$\frac{\partial^2 p}{\partial t^2} = \left(\frac{\beta}{\rho}\right)^2 \frac{\partial^2 p}{\partial x^2}$$ (2 pts).

2. Show how $\boldsymbol{v}$, the speed at which one pulse (or crest or trough, etc.) of the wave propagates, can be expressed in terms of β and , ρ , as per your finding in (a). Show reasoning, not just a final answer: Specifically, YOU CANNOT ASSUME that $v = \frac{\omega}{k}$. This is precisely what you can and must derive from definitions (4 pts).

3. Given all the above, and GIVEN ONE PARTICULAR MEDIUM, what happens to a wave when the disturbance producing it (like a hand shaking) occurs with TWICE THE FREQUENCY?

   Explain in at least a complete sentence and at least one equation, what happens to BOTH :

   the ANGULAR WAVENUMBER and SPEED of the wave (4 pts).

4. Using units/dimensional analysis as a guide, and analogies with transverse waves as further support, describe the quantity measured by β. It is some constant physical property of the medium, no? Why is this first time such property is appearing as relevant?

SCORE: _______________

D. **Point** (10 pts).

1. Assume that sound travels at ***a speed of 340 m/s relative to air***.

   A car is traveling on a non-windy day. It blares a horn. The car driver is distracted. She does not listen and has no idea what she hears.

   The car moves toward you at
   a speed of 44 m/s relative to air.

   You hear that car make a sound and you measure its frequency to be 500 Hertz.

   If she were actually paying attention to that sound her own horn made,

   WHAT FREQUENCY WOULD THE CAR'S DRIVER MEASURE (5 pts)?

2. A violin string – tacked down to the violin at each end -- is plucked.
   It experiences standing waves in the simplest possible way:
   Each of the two string ends, being fixed to the violin, sits perpetually still,
   but other than these two NODES, no other nodes are found on the string.

   Under such conditions, the string emits a sound of frequency f_1 = 680 Hertz.
   Since it involves only two nodes,
   this frequency is the frequency of the FIRST HARMONIC.

   Assume that the string has a constant linear mass density of 2 kg/m and is held with 800 Newtons of tension.

   i. What is the FREQUENCY for the SECOND HARMONIC of this sound (3 pts)?

   ii. How many nodes are observed for the 2040 Hertz harmonic (2 pts)?

SCORE: ______________

Part II: The Dif. Eq. for S.H.O. (35 pts).

CHOOSE ONE (1): EITHER all of A OR all of B.

-- Any additional parts or whole of a second choice will not be marked; if both appear to be attempted, credit will be awarded to the one deemed most convenient to mark.

A. CHOICE (A) (includes all four parts below).

Consider the following differential equation:

$$\frac{d^2r}{dt^2} = -\left(\frac{Qq}{4\pi\varepsilon_0 mR^3}\right) r,$$

in which Q, q, R, m and ε_0 are all non-zero constants.

i. Show that $r = \frac{1}{2}at^2 + v_0 t$ (in which a and v_0 are constants; $a \neq 0 \neq v_0$) ***IS*** **NOT** ***a solution*** to the differential equation presented above.

Your response must be largely *mathematical* yet include at least one clear thought expressed in *English* (7 pts).

ii. Show that $r = r_0 \cos(\omega t + \kappa\chi)$ (in which ω, κ, r_0 *and* χ are constants) ***IS a solution*** to the *differential equation* above (4 pts).

a. according to this solution, how is ω *related to* the *constant* term(s) in the DIFFERENTIAL EQUATION (4 pts)?

b. according to this solution, in what units must the constant product $\kappa\chi$ be measured (3 pts)?

Assume $\kappa\chi = 0, r_0 = 1$ and $\frac{Qq}{4\pi\varepsilon_0 mR^3} \approx 9 \times 10^9$ s^{-2}.

c. In seconds, what is the ***period of oscillation*** for the particle described by the above differential equation (7 pts)?

d. find the approximate value for r when t = *2 SECONDS* (7 pts)? CAREFUL.

e. If the value of the constant ε_0 were to triple (while all else remained fixed), then ***what would happen*** to the ***number of oscillations*** we would expect to count ***per hour***? CAREFUL.
(Would the number increase or decrease; by **precisely how much**?) (3 pts.)

B. CHOICE (B) (includes all four parts below) (35 pts).

SCORE: ________________

Near the surface of Earth, a particle of mass $\boldsymbol{m}$ is dangled from a long string, length $\boldsymbol{L}$; the particle oscillates along a small arc according to the differential equation

$$\frac{\partial^2 \theta}{\partial t^2} = -(20)\theta,$$

where $\boldsymbol{\theta}$ refers to an angular displacement measured from the vertical and $\boldsymbol{t}$ refers to time.
The particle's mass is given by $m = 8\ kg$.
The length of the string, L, is constant and was accidentally not recorded by the researchers – but can be deduced from all the other given information.
Whenever the particle arrives at a location of $\theta = .2\ radians$ from the vertical, the particle has no instantaneous speed. On both sides of the vertical, that is, $\theta = .2\ radians$ is repeatedly observed to be a 'turning point' for the particle's periodic motion.

i. Draw a clear diagram of this particle at some arbitrary point during oscillation, making sure to label variables and constants described above (4 pts).

ii. In what ***units*** should the constant (20) be measured (3 pts)?

iii. Approximating to three significant digits,
what is the ***angular frequency*** of this oscillator on a string (4 pts)?

iv. ***What is the particle's approximate SPEED at $t = 3$ seconds*** (4 pts)?

v. Assume that an experimenter begins measuring time at the instant the particle reaches .2 radians from the vertical. Assume, further, that the only force doing work on this dangling particle is gravity.

(a) ***How much Potential Energy*** does the particle have at $t = 0$ – ***immediately at the start*** (10 pts)?

(b) Assume that all the above values remain precisely the same, EXCEPT that the ***turning*** points at +/- .2 radians now occur at +/- .6 radians: In other words, the amplitude has been TRIPLED.

Given this particular change, describe what will happen to the value of the ***period*** of time for one cycle (10 pts).

SCORE: _______________

Part III: THE PUDDING (45 pts)

CHOOSE ***ONE*** STATEMENT FROM THE TWO OFFERED BELOW AND THEN,

in 2-Column (*Claim/Justification*) Format, beginning

1) FIRST, PROVIDE A CLEARLY LABELED DIAGRAM which presents the situation, clarifies many of the situational 'givens', and defines constants and variables to be mentioned in your derivation.

2) THEN with fundamental principles of physics, geometry, calculus and English,

DERIVE THE STATEMENT you choose. That is, show how the statement follows from a series of steps – each of which can be justified as a small and reasonable inference from some step before.

EITHER:

1. **If a long string of essentially uniform density (μ) is fixed at both ends into a tight and straight orientation (for which the string tension is known as T and the unperturbed shape of the string is known as the x-axis), then a quick disturbance (or 'pluck') to some small section of this string allows for the periodic motion of multiple disturbances along the string; under such motion, all points associated with the string can be related in space and time by the Differential Equation:**

$$\frac{\partial^2 y}{\partial t^2} = \left(\frac{T}{\mu}\right)\frac{\partial^2 y}{\partial x^2}.$$

***** OR *****

(next page. . .)

SCORE: ______________

To prove: **"The motion of a small-angle, planar pendulum is approximately Simple Harmonic".**

For this second choice, You May ASSUME:

1) A particle of ***m*** is suspended at the end of a long, light string, length ***L***;

2) It is held still at a small angular displacement from an equilibrium position –
found at the lowest possible point on the vertical;

3) the small angular displacement is called $\boldsymbol{\theta}$.

4) It is then released – with no starting speed at all – from the small angular displacement, called $\boldsymbol{\theta}$.

5) You may also assume:
a) Newton's 2nd Law,
b) The Small-Angle (Radian) Approximation for the Sine of an Angle,
c) Anything we have already established about the solution to the 2nd order Dif. Eq. for SHO.

(So please first write all the above into your sheet as "Givens"…)

SCORE: _______________