(MidTerm #1, Take2) The Quest:

Oscillation, Interaction & Superposition

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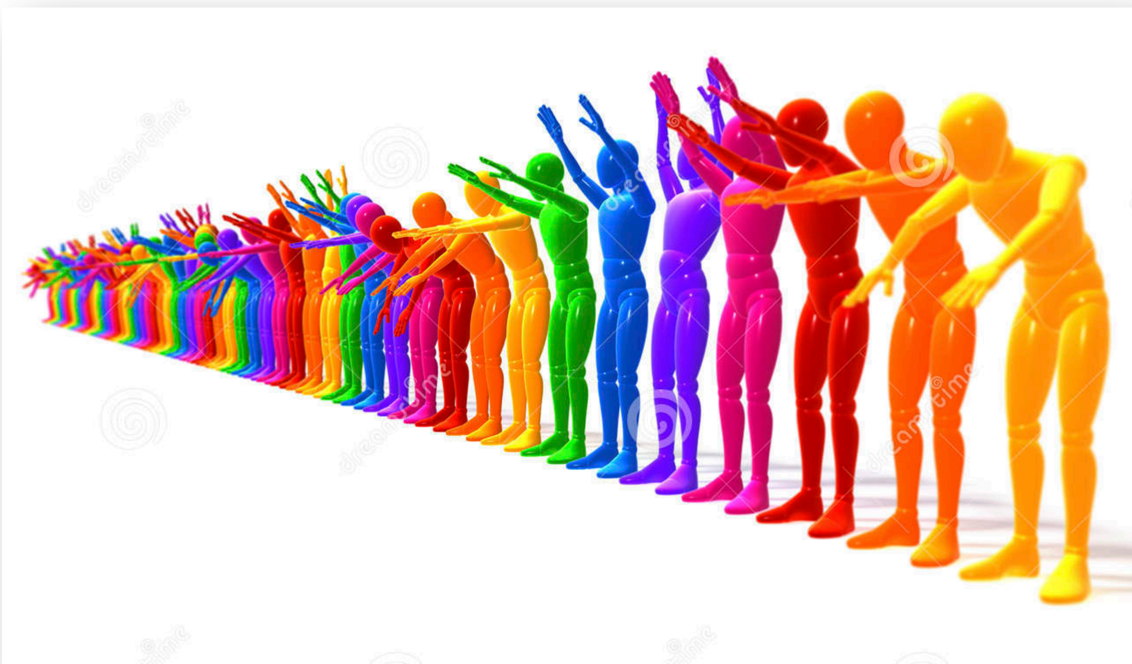
Wednesday, July 8, 2015



Name: ____

Section: ____________

SCORE: _______________

THE FOLLOWING RELATIONS UNDERLIE THE MATERIAL:

1) $\sum \vec{F} = m\vec{a}.$

2) $F = -Kx.$

3) $x = Acos(\omega t + \phi).$

4) $\omega \equiv 2\pi f.$

5) $f \equiv \frac{1}{T}.$

6) $\theta \equiv \frac{x}{r}.$

7) $\lim_{\theta \to 0} \frac{sin\theta}{\theta} = 1.$

8) $\frac{\partial^2 y}{\partial t^2} = \left(\frac{T}{\mu}\right)\frac{\partial^2 y}{\partial x^2}.$

10) $y = Acos(\omega t - kx).$

11) $k \equiv \frac{2\pi}{\lambda}.$

12) $v = \frac{\omega}{k}.$

13) $\vec{F} = -\frac{GMm}{r^2}\hat{r}.$

14) $G \approx 6.67\ x\ 10^{-11}\ \frac{Nm^2}{kg^2}$

15) $\mu \equiv \lambda \equiv \frac{dm}{dx}.$

16) $\sigma \equiv \frac{dm}{dA}.$

17) $\rho \equiv \frac{dm}{dV}.$

SCORE: ________________

Part I: **Exercises** (25 pts)

CHOOSE EITHER (A) or (B). Do ALL of ONE.

A. **Fall** (15 pts).

Consider this description of IO, one of Jupiter's four prominent moons:

$$\bar{F} = -\frac{GMm}{r^2}\hat{r}; G \approx 6.67 \times 10^{-11}\,\frac{Nm^2}{kg^2}$$

$$R_i \approx 1.82\times 10^6\ meters; M_i \approx 8.93 \times 10^{22} kg$$

(Also recall the corresponding data for Earth:

$$(Re \approx 6.37 \times 10^6\ meters; Me \approx 5.97 \times 10^{24}\ kg)$$

b. In meters/sec^2, to three significant digits, compute the approximate free-fall acceleration ($\boldsymbol{g}_i$) of any mass dropped very near IO's surface (3 pts).

c. Now assume that a mass is again to be dropped toward the center of IO, but this time within a tiny and frictionless tunnel carved out from the moon's center—and toward a location ONE-TENTH of the radius away from center.

At this height, what is the initial value for ***g(r=Ri/10)*** (i.e. what happens to the value you computed in (a), above) (3 pts)?

d. How many times greater or smaller is the Earth's radius – as compared to Io's radius? For example, if Object1's radius were 5 meters and Object2's radius were 10 meters, your answer would be something along the lines: "Object2'sRadius = $2X$ Object1'sRadius" or "Object1'sRadius = $\frac{1}{2}X$ Object2'sRadius" (2 pts).

SCORE: _______________

e. Assume that a mass were to be dropped from high above the surface of Io: so high that the height AS MEASURED FROM Io's SURFACE is precisely equal to Earth's RADIUS (as given above).

   At this Earth-height ABOVE IO's SURFACE, how does the new instantaneous value for g_i compare to the standard surface value (found in (a)?
   NOTE! Let this new value for g be called ***g(iSeC)) and solve for*** $\frac{g(iSeC)}{g_i}$ (3 pts)!

f. Assume that way out in the L'Engle Galaxy, there is a celestial object called Tesseract B-26 with PRECISELY DOUBLE (2X) THE DENSITY as that of IO.

   If Asteroid B-26 has a radius that is 3x as large as that of IO, how does g_{B26} on Asteroid B-26 compare to g_i on Io (how many times greater or smaller) (4 pts)?

SCORE: ______________

B. **Spring** (15 pts).

Consider two possible descriptions for the behavior of some system, below:

i)

$$\frac{\partial^2 I^2}{\partial t^2} = -(LC)I^2;$$
$$LC = 3.14 \times 10^{-5}\,\text{sec}^{-2};$$
$$I^2{}_0 = 300\ milli - Amperes^2.$$

and

ii)

$$\frac{\partial^2 I}{\partial t^2} = -(\omega^2)\,t;$$
$$\omega^2 = 3.14 \times 10^{-5}\,\text{sec}^{-2};$$
$$I_0 = 300\ milli - Amperes$$

a. Assuming that in each one, ***t*** stands for time
   (as measured from some initial moment called 0),

   indicate WHICH ONE of the above descriptions could refer to a system behaving in Simple Harmonic Oscillation; simply reply **i** or **ii** (3 pts).

b. To the one you decided was NOT an SHO, make the smallest possible correction you can—so as to turn it into a proper SHO. (Then leave it alone and answer all remaining questions with reference to the one you decided was SHO) (2 pts).

c. For whichever expression you DID CHOOSE to be an SHO, find its ***period*** of oscillation; provide a *number*, measured in seconds (5 pts).

d. For whichever expression you DID CHOOSE to be an SHO, determine how much current there is at t = 3 seconds from the beginning of measurement. That is, in milli-Amperes, determine the value of ***I*** at t = 3 (5 pts).

SCORE: ________________

CHOOSE EITHER (C) or (D):

C. **Wiggle** (10 pts).

Consider a particle [redacted]

$y = A\cos($ [redacted] , whe[redacted] x and y are variables and everything else is a constant.

a. Show that th[redacted] to the second order differential equation

$$\frac{\partial^2 y}{\partial t^2} = ($$ [redacted] (2 pts).

b. Show how v, [redacted] ugh, etc.) of the wave propagates, ca[redacted] finding in (a). Show reasoning, no[redacted] T ASSUME that $v = \frac{\omega}{k}$. This is pre[redacted] tions (4 pts).

c. Given all the [redacted] DIUM, what happens to a wave when [redacted]) occurs with TWICE THE FREQU[redacted]

Explain in at least [redacted] on, what happens to BOTH :

the ANGULAR WAVENUMBER and SPEED of the wave (4 pts).

SCORE: ________________

D. **Point** (10 pts).

$$\vec{A} \equiv 12\hat{i} - 5\hat{j} - 5\hat{k}$$
$$\vec{B} \equiv -5\hat{k}$$

i. Find $\vec{A} \cdot \vec{B}$ (3 pts).

ii. Find $\vec{A} \times \vec{B}$ (3 pts).

iii. Find $\overrightarrow{\mathbf{A}} \cdot (\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}})$ (4 pts).

SCORE: ________________

Part II: **The Dif. Eq. for S.H.O.** (35 pts).

CHOOSE ONE (1): EITHER all of A OR all of B.

-- Any additional parts or whole of a second choice will not be marked; if both appear to be attempted, credit will be awarded to the one deemed most convenient to mark.

A. CHOICE (A) (includes all four parts below).

Consider the following differential equation:

$$\frac{d^2r}{dt^2} = -\left(\frac{Qq}{4\pi\varepsilon_0 mR^3}\right) r,$$

in which Q, q, R, m and $\boldsymbol{\varepsilon_0}$ are all non-zero constants.

i. Show that $\boldsymbol{r} = \frac{1}{2}\boldsymbol{a}\boldsymbol{t}^2 + \boldsymbol{v_0}\boldsymbol{t}$ (in which $\boldsymbol{a}$ and $\boldsymbol{v_0}$ are constants; $\boldsymbol{a} \neq \boldsymbol{0} \neq \boldsymbol{v_0}$) ***IS*** **<u>NOT</u>** ***a solution*** to the differential equation presented above.

Your response must be largely *mathematical* yet include at least one clear thought expressed in *English* (7 pts).

ii. Show that $\boldsymbol{r} = \boldsymbol{r_0}\, cos(\omega t + \kappa\chi)$ (in which $\boldsymbol{\omega}, \boldsymbol{\kappa}, \boldsymbol{r_0}$ ***and*** $\boldsymbol{\chi}$ are <u>constants</u>) ***IS a solution*** to the *differential equation* above (4 pts).

a. according to this solution, how is $\boldsymbol{\omega}$ *related to* the *constant* term(s) in the DIFFERENTIAL EQUATION (4 pts)?

b. according to this solution, in what units must the constant product $\boldsymbol{\kappa\chi}$ be measured (3 pts)?

Assume $\boldsymbol{\kappa\chi} = \boldsymbol{0}, \boldsymbol{r_0} = \boldsymbol{1}$ and $\frac{Qq}{4\pi\varepsilon_0 mR^3} \approx 9 \times 10^9$ s^{-2}.

c. In seconds, what is the ***period of oscillation*** for the particle described by the above differential equation (7 pts)?

d. find the approximate value for $\boldsymbol{r}$ when $\boldsymbol{t} =$ ***7 full periods*** (7 pts)?

e. If the value of the constant R were to triple (while all else remained fixed), then ***what would happen*** to the ***number of oscillations*** we would expect to count ***per hour***?
(Would the number increase or decrease; by **precisely how much**?) (3 pts.)

SCORE: ______________

B. CHOICE (B) (includes all four parts below) (35 pts).

A particle of mass $\boldsymbol{m}$ is dangled from a long string, length $\boldsymbol{L}$; the particle oscillates along a small arc according to the differential equation

$$\frac{\partial^2 \theta}{\partial t^2} = -(\mathbf{9.81})\boldsymbol{\theta},$$

where $\boldsymbol{\theta}$ refers to an angular displacement measured from the vertical and $\boldsymbol{t}$ refers to time.

The particle's mass is given by $\boldsymbol{m} = .080\ \boldsymbol{kg}$.

The length of the string, L, is constant and was accidentally not recorded by the researchers – but can be deduced from all the other given information.

Whenever the particle arrives at a location of $\boldsymbol{\theta} =. \mathbf{350}\ \boldsymbol{radians}$ from the vertical, the particle has no instantaneous speed. On both sides of the vertical, that is, $\boldsymbol{\theta} =. \mathbf{350}\ \boldsymbol{radians}$ is repeatedly observed to be a 'turning point' for the particle's periodic motion.

i. Draw a clear diagram of this particle at some arbitrary point during oscillation, making sure to label variables and constants described above (4 pts).

ii. In what ***units*** should the constant (9.81) be measured (3 pts)?

iii. Approximating to three significant digits,
what is the ***angular frequency*** of this oscillator on a string (4 pts)?

iv. ***How much time passes*** between some moment the particle is found at the equilibrium until the soonest (next) moment at which the particle has returned to equilibrium (4 pts)?

v. Assume that an experimenter begins measuring time at the instant the particle reaches .350 radians from the vertical. Assume, further, that the only force doing work on this dangling particle is gravity.

(a) ***How much Potential Energy*** does the particle have at $\boldsymbol{t} = \boldsymbol{1}$ ***second after*** this clock starts (10 pts)?

(b) H***ow much Kinetic Energy*** does the particle have at $\boldsymbol{t} = \boldsymbol{1}$ ***second after*** this clock starts (10 pts)?

SCORE: _______________

Part III: **THE PUDDING** (45 pts)

CHOOSE ***ONE*** STATEMENT FROM THE TWO OFFERED BELOW AND THEN,

in 2-Column (*Claim/Justification*) Format, beginning

1) FIRST, PROVIDE A CLEARLY LABELED DIAGRAM which presents the situation, clarifies many of the situational 'givens', and defines constants and variables to be mentioned in your derivation.

2) THEN with fundamental principles of physics, geometry, calculus and English,

DERIVE THE STATEMENT you choose. That is, show how the statement follows from a series of steps – each of which can be justified as a small and reasonable inference from some step before.

EITHER:

[redacted]nsity (μ) is fixed at [redacted]n (for which the [redacted]ring tension is known as T and t[redacted]rturbed shape of [redacted]e string i[redacted]nown as the x-axis), [redacted]quick disturbance [redacted]r ´pluck) [redacted]e small[redacted]ction of [redacted]ing allows for the [redacted]riodic m[redacted]mul[redacted]disturba[redacted]g the string; [redacted] such motion, all points a[redacted] the string can be [redacted]lated in space and time by the Diff[redacted]Equation:

[redacted]

*** OR ***

(next page. . .)

SCORE: _______________

2. If a [redacted]ed by a displ[redacted]dense RIN[redacted]r which the total mass is M, a[redacted]which the radiu[redacted]isplac[illegible]t, $\vec{R}$, is perfectly pe[redacted]ular to the sepa[redacted]on $\vec{r}$, the[redacted]RIN[illegible] M exer[redacted]t gravi[redacted]al for[illegible]e o[illegible] the[illegible]artic[illegible] of

[redacted]

A number of more p[redacted]symbolically writte[redacted]might BE VISIBLE ON THE WHITEBOARD, [redacted]y add to that list if[redacted]e statements are truly implicit in the situation.

SCORE: _______________