VIRTUAL CLASS:
Monday, March 20, 2017

We do not physically meet today.
Instead, please take the following seriously and submit electronically – via 'Submit' button.
Of course, you are encouraged to work together, etc.:

I. The First Five of the steps below are numbered and are GIVEN.

II. They refer to a diagram found at
http://www.yaverbaum.org/JohnJay/Classes/Lecture.204/Exams/MT2_s17/WaveEqDerive/WaveEq.png

Anything stated on or about that diagram on that sheet is also given.

III. For EACH AND EVERY STEP (of the derivation for the Wave Equation):

A. Number the step – starting fresh from 1.

B. In a COMPLETE THOUGHT in terms with which YOU ARE COMFORTABLE, provide a QUICK BUT COMPLETE JUSTIFICATION/REASON/EXPLANATION for EACH STEP.

C. Number accordingly:
If you are explaining step 1, then number your explanation with a 1, etc.

D. COMPLETE THOUGHT = Either EQUATION or COMPLETE SENTENCE of ENGLISH.
One exception = "Definition." But there, you MUST SAY "Definition of ___________________"
and compete blank.

E. Many reasons will be super simple like "both sides of eq. (_) divided by 2", but even that must be written like that – not "algebra".

IV. Identify EVERY Step where a fresh/independent APPROXIMATION occurs (even though the equal sign cannot get any more squiggly)

V. Identify where/how all the approximation turns back into strict equality.

VI. In 2 – 5 complete sentences of your own English words:

A. What does this conclusion mean? What IS a wave according to it?

B. Given the 'Givens', what is the MEANING/IMPLICATION of this derivation? Why do we care, in this particular context, that the end can be concluded from the beginning?

$$1)\quad \mu \equiv \frac{\Delta m}{\Delta x}$$

$$(2)\quad y = f(x,t):$$

$$(3)\quad \frac{\Delta t}{t}, \frac{\Delta x}{x}, \frac{\Delta \theta}{\theta} \to 0$$

$$(4)\quad \lim_{x \to -\infty} = 0 = \lim_{x \to \infty}$$

$$(5)\quad \sum \Delta \vec{F} = \Delta m \vec{a}$$

$$\vec{T}_2 - \vec{T}_1 = \Delta m \vec{a}$$

$$T_{2x} - T_{1x} = \Delta m a_x$$

$$T_{2x} - T_{1x} = \Delta m \cancel{a_x} \leftarrow 0$$

$$T_{2x} = T_{1x}$$

$$T_{2y} - T_{1y} = \Delta m a_y$$

$$T_2 \sin\theta_2 - T_1 \sin\theta_1 = \Delta m a_y$$

$$T_2 \equiv \| \vec{T}_2 \| = \| \vec{T}_1 \| \equiv T_1$$

$$\text{Let } T \equiv T_2 = T_1$$

$$T(\sin\theta_2 - \sin\theta_1) = \Delta m a_y$$

$$\lim_{\Delta x \to 0} \left(\frac{\sin\theta}{\theta} \right) = 1$$

$$T(\sin\theta_2 - \sin\theta_1) = \Delta m a_y$$

$$T(\theta_2 - \theta_1) \approx \Delta m a_y$$

$$\lim_{\Delta x \to 0}\left(\frac{\tan\theta}{\theta}\right) \approx 1$$

$$T(\theta_2 - \theta_1) \approx \Delta m a_y$$

$$T(\tan\theta_2 - \tan\theta_1) \approx \Delta m a_y$$

$$T\left(\frac{\sin\theta_2}{\cos\theta_2} - \frac{\sin\theta_1}{\cos\theta_1}\right) \approx \Delta m a_y$$

$$T\left(\frac{L\sin\theta_2}{L\cos\theta_2} - \frac{L\sin\theta_1}{L\cos\theta_1}\right) \approx \Delta m a_y$$

$$T\left(\frac{L\sin\theta_2}{L\cos\theta_2} - \frac{L\sin\theta_1}{L\cos\theta_1}\right) \approx \mu \Delta x a_y$$

$$T\left(\left.\frac{\Delta y}{\Delta x}\right|_{x+\Delta x} - \left.\frac{\Delta y}{\Delta x}\right|_{x}\right) \approx \mu \Delta x a_y$$

$$T\left(\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}\right) \approx \mu \Delta x a_y$$

$$T\left(\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}\right) \approx \mu \Delta x \frac{d^2y}{dt^2}$$

$$\frac{\cancel{T}\left(\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}\right)}{\Delta x} \approx \frac{\mu \cancel{\Delta x}}{T}\frac{d^2y}{dt^2}$$

$$\frac{\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}}{\Delta x} \approx \frac{\mu}{T}\frac{d^2y}{dt^2}$$

$$\frac{\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}}{\Delta x} \approx \left(\frac{\mu}{T}\right)\frac{d^2y}{dt^2}$$

$$\frac{\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}}{\Delta x} \approx \frac{\mu}{T}\frac{d^2y}{dt^2}$$

$$\frac{\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}}{\Delta x} \approx \left(\frac{\mu}{T}\right)\frac{d^2y}{dt^2}$$

$$\text{Let } y' \equiv f'(x) \equiv \frac{dy}{dx}$$

$$\frac{f'(x+\Delta x) - f'(x)}{\Delta x} \approx \left(\frac{\mu}{T}\right)\frac{d^2y}{dt^2}$$

$$\lim_{\Delta x \to 0}\left[\frac{f'(x+\Delta x) - f'(x)}{\Delta x}\right] = \lim_{\Delta x \to 0}\left[\left(\frac{\mu}{T}\right)\frac{d^2y}{dt^2}\right]$$

$$\cdots \quad \frac{d}{dx}(f'(x)) = \left(\frac{\mu}{T}\right)\frac{d^2y}{dt^2}$$

$$\frac{d}{dx}y' = \left(\frac{\mu}{T}\right)\frac{d^2y}{dt^2}$$

$$\frac{dy'}{dx} = \left(\frac{\mu}{T}\right)\frac{d^2y}{dt^2}$$

$$y' \equiv \frac{dy}{dx}$$

$$\left(\frac{d^2y}{dx^2}\right) = \left(\frac{\mu}{T}\right)\frac{d^2y}{dt^2}$$

$$\frac{\partial^2y}{\partial x^2} = \left(\frac{\mu}{T}\right)\frac{\partial^2y}{\partial t^2}$$

$$\boxed{\frac{\partial^2y}{\partial t^2} = \left(\frac{T}{\mu}\right)\frac{\partial^2y}{\partial x^2}}$$