Standing Waves.

A violin string – tacked down to the violin at each end -- is plucked. It experiences standing waves in the simplest possible way: Each of the two string ends, being fixed to the violin, sits perpetually still,
but besides these two NODES, no other nodes are found on the string.

Under these conditions, the string emits a sound of frequency $f_1 = 680$ Hertz.
Since it involves only two nodes,
this frequency is the frequency of the FIRST HARMONIC.

Assume that the speed at which sound travels through the string is given as $v_{ws} = 500$ m/s.

i. What is the FREQUENCY for the SECOND HARMONIC of this sound?

ANSWER: $f_2 = 1360$ Hertz.

Some explanation: A full wavelength is one total crest and one total trough (or the equivalent).

L stands for the length of the string, and the string is bounded at both ends. Each end of the string MUST sit still – at the equilibrium 'height'. This constrains the set of possibilities.

So we can fit one crest on the string or one crest and one trough or two crests and a trough, etc. In general, any integer multiple of half-wavelengths can be fit on the string – that allows for an infinite set of possible harmonics. Each higher frequency is called the next harmonic. They are all integer multiples of the very lowest (simplest) – known as harmonic #1 or the fundamental frequency. This fundamental frequency is characterized by 2 nodes. It can fit only a crest or only a trough at once. Like jumprope.

The next (second) harmonic has a node right in the middle – three equidistant nodes. This is a crest and trough at the same time – alternating.

The next (third) has a total of four nodes – equally spaced from each other and equally spaced from the ends. It fits crest trough crest or trough crest trough (etc), i.e. : a wavelength and a half or 3/2 wavelengths.

This pattern of fitting half wavelengths continues infinitely. A string can, in principle, fit infinity nodes. Each next harmonic is another equally spaced node. Necessarily, therefore,

The number of nodes = n (harmonic number) + 2

But infinity does not mean everything imaginable. ONLY integer multiples of half-wavelengths can fit (for standing waves). If we let *n* stand for the 'harmonic number', then

$$L = \frac{n\lambda}{2}.$$

This relation defines the infinite set of possible frequencies at which standing waves can travel along this string.

ii. What is the *wavelength* of the third harmonic on this string?

Answer: $\lambda_3 = \frac{v}{f_3}$

and, if you look at the original (key) statement about fitting half-wavelengths on L, then you'll soon see how it must be the case that $f_3 = 3f_1$. So...

$$\boldsymbol{\lambda_3 = \frac{500}{2040} \approx .245\ m}$$

iii. A higher harmonic for this sound has a frequency of 3,400 Hertz. How many nodes does this harmonic have (3 pts)?

Well, n = 5 (see above).

Always, number of nodes = n + 1.

So, answer:

6 NODES.

iv. Can this string produce standing waves at a frequency of 4000 Hertz? If not, why not? If so, what harmonic is it?

No. 4000 is NOT an integer multiple of 680. Can't fit.