Unspringy Springs

(aka: *The Gravitational Tunnel*, aka: *Subway Line 42*)

Physics 204,
John Jay College of Criminal Justice, the CUNY
Daniel A. Martens Yaverbaum, Max Bean

Given that:

1) '***UNIFORM DENSITY***' means that the ratio of mass to volume is a constant.

   Within a region of uniform density, M/V is the same, no matter what chunk of space (V) we choose to look at.

2) A comparatively small object ('***particle***') of mass ***m*** placed in anywhere within a spherical SHELL of substantially larger mass (say, ***M***) will experience a NET Gravitational Force of ZERO.

Problem 1

Imagine a vast solid sphere of uniform density floating somewhere in space. Maybe it's a planet. Maybe it's a moon. Maybe it's a giant floating bowling ball.

Imagine that, somewhere ***inside this sphere*** is a particle.

It may help to image that this particle is not attached to the rest of the sphere. Maybe the particle is in a cave deep beneath the surface or a very deep hole. This is not strictly necessary, but it will help you picture the situation.

The giant solid sphere has mass ***M*** and a radius ***R***.
The particle has mass ***m*** and a negligible radius.
The particle is a distance ***r*** from the center of the giant solid sphere.
If you have a clear understanding of the scenario, you already know that $r < R$.

Derive an equation that gives F_{gr}, the gravitational force that the giant sphere exerts on the particle, as a function of r, m, M, and R.

Problem 2

Imagine that earth is a giant sphere of uniform density. (This is not actually true. Earth gets much denser as you go towards the center, but in physics we like to keep things simple.)

Imagine that a tunnel is dug from some point on earth's surface, straight through the center of earth to a point on the opposite side of earth. In other words, the tunnel is a diameter of earth.

Imagine that a small object of mass ***m***, such as a boulder or a subway car, is dropped (from rest) into the tunnel.

Assume that all friction (even air resistance) is negligible. Do not worry about earth's rotation. Just imagine it's a uniform, frictionless sphere floating in space.

As you work on the questions below, ***use your answer to problem 1*** on the previous page. This is where all the work you did there becomes useful.

a. What happens to the ***speed*** of the object as it falls ***towards*** the center of earth?
b. What is the acceleration of the object the moment after it is dropped?
c. What happens to the acceleration of the object as it falls ***towards*** the center of earth?
d. What is the acceleration of the object when it reaches the center of earth?

The object flies past the center of earth and continues towards the far end of the tunnel.

e. What happens to its acceleration as it moves from the center towards the far end of the tunnel?
f. What happens to its speed as it moves towards the far end of the tunnel?
g. Will it get all the way to the far end of the tunnel? Will it stop before it gets there? Will it go past the far end of the tunnel and fly out above ground? Explain/justify?
h. If nothing interferes with the motion of the object, what will happen over time?

And now, the million dollar question:

i. In terms of ***G***, ***M*** (of earth), and ***R*** (of earth), *how much time will it take the object to reach the other end of the tunnel*?

   HINT: Go back to your $\mathbf{F}_{\mathbf{gr}}$ equation from problem 1 on the previous page.
   Apply Newton's 2nd Law and solve for acceleration.
   Does this situation look familiar? Is acceleration a function of position?
   What kind of motion is this????? (It begins with an S… H………)

j. Given that $M_E = 6\times10^{24}kg$, $R_E = 6.4\times10^{6}m$, and $G = 6.67\times10^{-11}\frac{Nm^2}{kg^2}$
   Evaluate your answer to (i) above to find an actual number of minutes & seconds that it will take the object to reach the other side of earth.

Problem 3

Now imagine again that a tunnel is carved out from one surface location on earth to another surface location. This time, however, the tunnel is a chord of arbitrary length. It need not pass through Earth's center.

How much time elapses as a mass travels from one side to the other of an arbitrarily long gravitational tunnel?

Hint: Again, think hard about what simple harmonic oscillation really means.