Resonance & Volume - Solutions

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I. Wave-Interference Simulation

Answer the following questions:

A. What does the resulting interference-pattern look like when the red & blue waves are exactly out of phase?

Total negative interference: black line is flat

B. What does the interference-pattern look like when the when the red & blue waves are perfectly in-phase?

Maximum positive interference: peaks are at their highest, troughs at their lowest.

C. What do you notice about the nodes?

These points in the resulting wave do not move: they stay at zero.

D. What do you notice about the anti-notes?

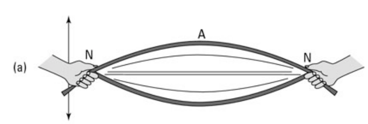
The points at the antinodes have the maximum amplitude for their oscillation.

II. Standing Waves & Harmonics

A. 1-D: A string 30 cm long with a linear mass density of .65 g/m (6.5 x 10^{-4} kg/m) and subjected to 45 Newtons of tension.

1. Find the *fundamental frequency* for the string.
2. Find the *3rd harmonic* for the same string.

"Fundamental frequency": first harmonic.
First harmonic just looks like this:



That's half of a wave, and its 30 cm, so a whole wave is 60 cm = 0.6 m.
So $\lambda = 0.6\,m$. If we find v, we can solve for *f*.
From the wave equation for a transversal wave on a string, we know that $v = \sqrt{\frac{T}{\mu}}$ (where T is tension).

$$v = \sqrt{\frac{45}{0.00065}} = 263\,m/s$$

$$v = \lambda f$$

$$263 = (0.6)f$$

$$f = 263/0.6 = 438\,Hz$$

B. 2-D: Two speakers are placed 5 m apart. Both play an identical 440 hz tone at the same volume, in phase with each other. Assume that the speed of sound in air is 340 m/s.

Find the two points (locations) closest to the mid-point between the speakers at which a tiny microphone could be placed and perpetually pick up (detect, measure) nothing but silence.

i.e.: Find the *nodal* locations nearest to the *central antinode*.

At central antinode, path fr speaker 1 (L_1) = path from speaker 2 (L_2) = 2.5 m.
(because the speakers are 5 m apart, and the CA is right between them.)
Therefore waves arrive perfectly ***in*** phase and interference is constructive.
Node occurs when waves are perfectly ***out of*** phase: 180° (1/2 a cycle) off.
So, at node, $|L_1 - L_2|$ = half a cycle (or 1.5 cycles, or 2.5 cycles, etc.).
If we move ¼ wavelength away from CA, towards speaker 2, then:
$\boldsymbol{L_1 = 2.5 + (0.25)\lambda}$ and $\boldsymbol{L_2 = 2.5 - (0.25)\lambda}$
So, $\boldsymbol{L_1 - L_2 = (2.5 + (0.25)\lambda) - (2.5 - (0.25)\lambda) = 0.5\lambda}$, so this is a node.
Of course, there is also a node on the other side of the CA, if we go ¼ cycle towards speaker 2 instead.

That was a kind of an intuitive guess-and-check. We could solve this more algebraically too.
Say we start at the CA and move a distance X towards speaker 2.
Now, $\boldsymbol{L_1 = 2.5 + x}$ and $\boldsymbol{L_1 = 2.5 - x}$
So, $\boldsymbol{\Delta L = L_1 - L_2 = (2.5 + x) - (2.5 - x) = 2x}$.
If we $\boldsymbol{\Delta L}$ to equal half a wavelength, we just substitute & solve:
$\boldsymbol{\Delta L = (0.5)\lambda = 2x}$
$\boldsymbol{x = \frac{(0.5)\lambda}{2} = (0.25)\lambda}$
If we want the next antinode over, we could set $\boldsymbol{\Delta L}$ equal to 1.5 $\boldsymbol{\lambda}$ instead.
Etc.

Now, of course, we still need a location, in meters, and for that we need to know $\boldsymbol{\lambda}$, which we can get quickly from speed and frequency:
$\boldsymbol{v = \lambda f}$
$\boldsymbol{\lambda = \frac{v}{f} = \frac{340}{440} = 0.77\ \mathrm{m}}$

So, if the nodes are a quarter wavelength on either side of the CA, that's 0.77/4 = 0.19 m to the left and right of the CA, or 2.5-0.19 = 2.31 m from each speaker.