OSCILLATIONS & WAVES

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I. THE GRAVITATIONAL TUNNEL (AKA: SUBWAY LINE 42)

The statements immediately to follow (even when long and complicated) are considered (in this context)

GIVEN.

You may assume and rely on them for the problem/proof to follow a bit further down.

Note: In some cases, 'GIVEN' might mean 'self-evident' or 'obvious', but in other cases, it might not. GIVEN might not mean 'obvious'; it can simply mean 'somehow established prior to this discussion'.

GIVEN (for this context) →

1) Planet Earth can be treated as one large SOLID SPHERE of UNIFORM (constant) DENSITY, with constant ('known') mass M, constant ('known') radius R.

2) Given ANY TWO POINT MASSES, say m_1 and m_2, then m_1 will necessarily exert a gravitational force onto m_2 and thereby pull m_2 directly toward m_1.

   (• • • If we wish to be careful and precise in keeping track of directions, we can do the following:
   Let $\vec{r}$ be a displacement vector that points directly from m_1 to m_2
   and is precisely as long as the distance between $m1$ and m2,
   then the customary scalar quantity r, referring to distance between masses,
   can be understood as the pure magnitude of the vector $\vec{r}$,
   while the possibly less familiar $\hat{r}$ is understood as the pure direction of $\vec{r}$ • • •)

   Hence, the gravitational pull exerted by m_1 onto m_2 is given by force given by:

$$\vec{\mathbf{F}} = -\frac{Gm_1 m_2}{r^2}\hat{r}$$

   This Claim is symmetric (applies identically to the pull exerted by m2 onto m1), but is always and only a statement about the interaction which arises BETWEEN 2 (infinitesimally small) *POINTS OF MASS* ('*particles*').

3) Interaction among anything larger or more complicated than points can be computed only by adding up the above results one point-pair at a time...

4) According to Chapter 13 of your textbook, a comparatively small object ('***particle***') of mass ***m*** placed in anywhere within a spherical SHELL of substantially larger mass (say, ***M***) will experience a NET Gravitational Force of ZERO. (!)

That is, each little piece of shell ***M*** makes contributes a very real and significant contribution of gravitational force to a large accumulation of force vectors all acting on ***m***. The particle ***m*** is being pulled from above, below, from the left, the right, etc.

Unless it happens to be at dead center, ***m*** is closer to some of these shell bits and farther from others. The gravitational force contributions coming in from all different locations on the shell, therefore, vary in size. But they also all pull in varying directions. The crazy thing, it turns out, is that all the varied directions and magnitudes perfectly cancel out one another -- so that the total gravitational pull experienced by any particle within the borders of the shell sums to... Nothing!

This result is HARDLY OBVIOUS and HARDLY EXPECTED. In fact, it's excitingly surprising to many of us, but it does turn out to be true. Always. And we will prove it a bit later in the course. For the purpose of the problem/proof right now at hand, you may simply assume it to be true. You will do yourself a great service if you try to maintain some sense of why/how the full cancellation could be plausible -- it makes it easier to picture the result itself -- but you need not strain to justify it precisely as yet. Simply know that the statement is true and relevant. For now. The 'given' statement, again, is; "The net gravitational force exerted by a spherical shell onto any particle within that shell... is ZERO".

5) The Universal Gravitational Constant,
simply to keep our measuring units all self-consistent, is given by :

$$G \approx 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}.$$

6) '***UNIFORM DENSITY***' means that the ratio of mass to volume is a constant:

That for any given segment of (3-D) space, ***V***, we expect to find an approximately unwavering amount of mass ***M***. $\frac{m}{v} = \frac{M}{V}$

So, in general, it is fair and useful to say: $\frac{M}{V} = \frac{m}{v}$.

By VOLUME, here, we mean the volume of a sphere:

$$V = \frac{4}{3}\pi r^3$$

Gravitation → $\frac{m}{v} = \frac{M}{V}$

a)

SO, given all the above,

Derive $\boldsymbol{F_{gr}} = \boldsymbol{f(r)}$,

for which F_{gr} stands for the net gravitational force (dependent variable) exerted on some particle of mass, ***m***, by all the bits of a large SOLID sphere of mass ***M***,

as determined by ***r***, displacement from the center of a solid sphere (independent variable) -- assuming that ***r =< R***, that is:

that the particle is located somewhere within the solid sphere.

b. Using the result you derived in (a), above,
imagine that a tunnel is carved out from one place on Earth's surface
to another place on Earth's surface.
Imagine that this tunnel passes through Earth's center so that it is a diameter.
A small mass ***m***, such as a boulder or a subway car,
is dropped into one end of this tunnel.
The tunnel is empty of anything frictional--including air.
Assuming that ***m*** free-falls through this gravitational tunnel,
the key question becomes:

How much time will it take for the mass to reach the other side?

i. Hint: This key question, above, is where all your work for (a) becomes worth it. This is where your understanding of simple harmonic motion becomes relevant. This is where you see why simple harmonic motion is such a sweet concept.

First, find this time as a general expression: as a function of the given and fundamental constants (***G, M, R***).

ii. Second, evaluate your function in order to get an actual numerical measurement -- an actual number of MINUTES, in this case, as an answer for time across.

In order to obtain a numerical answer for time, use the standard numerical values for Earth's characteristics:

$$M_E \approx 6 \times 10^{24}\,\textbf{kg}$$

$$R_E \approx 6.4 \times 10^{6}\,\textbf{meters}.$$

c. Now imagine that a tunnel is carved out from one surface location to another surface location.
This time, however, the tunnel is a chord of arbitrary length. It need not pass through Earth's center.
How much time elapses as a mass travels from one side to the other
of an arbitrarily long gravitational tunnel?!
Hint: Again, think hard about what simple harmonic oscillation really means.

Again, this is worth it. Again, it's not as difficult to solve as it may sound, Your answer will also include terms for Mm and R, but these are all constants.

In short, what happens to your WEIGHT as you travel from location to location WITHIN a solid (uniformly dense) sphere?

What is F_{gr} as a function of r ?

II. Text Problems: Regarding Wave Motion.

a) *Halliday, Resnick, Walker*, End of Chapter 16 (approx. p. 474: Problem 29.)

b) *HRW*, End of Chapter 16 (approx. p. 474: Problem 43.)

c) Note the derivation provided first thing section 16.4. Assume that you will be quizzed on this at some point soon. Practice it repeatedly on a blank sheet of paper. Be able to reproduce the derivation without looking at it. If you can find a simpler version of this derivation on the internet (that still makes use of second derivatives and Newton's 2nd Law), please feel free to learn that one instead. A physicist named David Griffiths wrote one for his electrodynamics text. If you can find that one, for example, all the power to you.

d) In three complete English sentences, explain the similarities and differences between the meanings of "equation 15-3" and "equation 16-45". Why do we need both?