In Pursuit of PUDDING

The Practice of Proof

Physics 204

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SUBMIT in 2-Column, Claim/Justification Form, an honest and thoughtful attempt at your own Derivation ("Proof") for each of the following statements:

REMEMBER: Every STEP will consist of a "CLAIM" (or 'statement') in the left-hand column accompanied by a "JUSTIFICATION" (or 'reason') in the right-hand column.

- All steps will be numbered; use one number for each one row consisting of one claim and one justification.

- Both parts of EVERY STEP must consist of COMPLETE THOUGHTS, i.e. COMPLETE STATEMENTS. If expressed in English, the statement will include a subject and a predicate. If expressed mathematically, the statement will include an equal sign and something on each side of it. If a diagram is somehow incorporated, terrific! – but then it must be labeled and the labels must then somehow lead to complete thoughts.

- Any and every statement that you consider both GIVEN (ASSUMED) and RELEVANT must be presented first as Steps. In other words, anywhere from two to six (or even more) of your first steps will be simple statements that we have already identified as 'fair game', such as "Average Speed is Defined as Distance per Time". The justification for each of such "GIVEN" steps is simply.. . . "GIVEN" .

- When you present justifications, use your own words. The reasons MUST MAKE SENSE to YOU. That's ultimately the point of all this. So your language need not be 'official', but it should be respectful of the ideas and expressive of actual reasoning – not games or gimmicks or mnemonics that you remember from when the big kids showed you black market arithmetic out in the alleyways of Long Island and you picked up expressions like "FOIL". "FOIL" is a really handy technique. It is not, however, a justification or explanation of any thought.

- A JUSTIFICATION is a REASON or EXPLANATION for why we believe something to be true. The belief you're defending was stated in a previous line. THEREFORE:

    - Whenever possible, refer to previous line numbers in your justifications, for example: "One side of an equation can be divided by m as long as the other side is also, line 8". (This is a big part of why we first include the GIVENS as legitimate steps.)

    - Justifications, therefore, are generally written in the PAST TENSE. They are about WHY IT WAS OK that something just happened. They are NOT about setting up excitement or motivation for something you feel is about to happen. That would be a "motivation."

Given the above, please attempt the following proofs.

The first proof you have already essentially done in one way or another at least once. You should now be at the stage of focusing intensely and personally on a few very specific steps and on precisely which aspects of them you do and do not understand. You should be picky about how you wish to express your justifications for these steps. You should start getting used to having board meetings with yourself – but somewhat quietly, without as many markers and deny it in public.

The second proof is a generalization of exercises we have done in lecture – that were intended to demonstrate its truth. Your goal is to try and turn those specific examples into a general conclusion that represents some kind of general reasoning. That's not necessarily as easy as it doesn't sound, but I have already seen what you – in particular – are capable of; it includes this. Even if not perfectly on the first shot.

I) "The motion of a small-angle, planar pendulum is approximately Simple Harmonic".

For this (familiar) first one, You May ASSUME:

1) A particle of $\boldsymbol{m}$ is suspended at the end of a long, light string, length $\boldsymbol{L}$;

2) It is held still at a small angular displacement from an equilibrium position – found at the lowest possible point on the vertical;

3) the small angular displacement is called $\boldsymbol{\theta}$.

4) It is then released – with no starting speed at all – from the small angular displacement, called $\boldsymbol{\theta}$.

5) You may also assume: a) Newton's 2nd Law,
b) The Small-Angle (Radian) Approximation for the Sine of an Angle,
c) Anything we have already established about the solution to the 2nd order Dif. Eq. for SHO.

2) The Net Gravitational Force exerted by a uniform RING (total Mass M, radius $\vec{R}$) onto a point mass (at a displacement of $\vec{r}$ from the ring's center) can accurately be described as

a. $\vec{F} = -\frac{GMmr}{(r^2 + R^2)^{3/2}}\hat{r}$ if $\hat{r} \times \hat{R} = 1$,

b. $\vec{F} \neq -\frac{GMmr}{(r^2 + R^2)^{3/2}}\hat{r}$ if $\hat{r} \cdot \hat{R} = 1$.

For this proof, you may assume:

1) $\vec{R}$ is a ("radial") vector that points from the center of the ring to any point on the ring itself (i.e. a vector whose size is the ring radius and whose direction is from center *out*);

2) $\vec{r}$ is a vector that points from the center of the ring to the point mass;

3) $\vec{r} > \vec{R}$;

4) The Law of Universal Gravitation is true (of point masses);

5) The principle of superposition is true;

6) the fundamental theorem of calculus is true. . .

. . . And, p.s., it's generally fair to assume the truth of anything else we've ever established or used in physics class/lab, such as the Pythagorean theorem -– unless we were just kidding or making a mistake or it was opposite day or it was all just a bad indigestion-induced dream . . .