Gauss's Law:
E-Fields for Continuous Charge Distributions

PHYSICS 204, YAVERBAUM, BEAN
JOHN JAY COLLEGE OF CRIMINAL JUSTICE, THE CUNY

Note: some of the challenges below were presented in lecture. Of course you can and should refer to your notes as you go, but don't go to your notes first. Force your brain try to reconstruct the solutions on its own. This is essential to developing a deeper understanding.

I. **Derivation**

DERIVE the Gauss Law result for the particular case of a single point charge.

That is, ***given***:

- ***Coulomb's Law*** ($F = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}$),
- ***The Definition of Electric Field*** ($\vec{E} = \frac{\vec{F}}{q}$), and
- ***The Definition of Electric Flux*** ($\Phi = \oint \vec{E} \cdot d\vec{A}$),

Show that the electric flux through a sphere of radius ***r*** surrounding a point charge of charge-magnitude ***q*** is equal to $\frac{q}{\epsilon_0}$. Do your best to justify key steps, e.g. pulling E out of the integral.

Assume that the result above can be generalized to all continuous charge distributions and all closed ("Gaussian") surfaces. In other words, for the rest of this sheet, assume ***Gauss's Law***.

II. **Cylindrical Symmetry**

Use Gauss's Law to calculate the following. Diagram required for *each* scenario.

a. The (magnitude of the) Electric Field at some distance ***r*** from the center axis of a *long* charged cylinder radius ***R*** made of ***conducting material***. The cylinder has linear charge density $\lambda = \frac{q}{l}$.
   i. Case 1: assume that ***r*** > R.
   ii. Case 2: assume that ***r*** < R.—the charge is ***inside*** the cylinder.

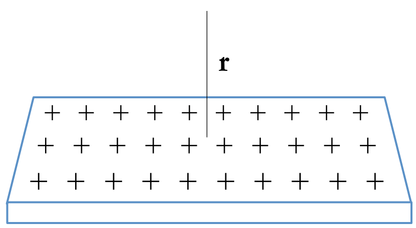
   Hint 1: you will need to draw your Gaussian surface inside the cylinder.
   Hint 2: If the cylinder is evenly distributed and symmetrical around your surface, the electric field through it must be evenly distributed (uniform) as well.

b. Same scenario as b above, but now the cylinder is made of ***insulating material***, and the charge is evenly distributed through the material. So instead of linear charge density, we'll find the field in terms of ***volumetric*** charge density $\rho = \frac{q}{V}$.
   i. Case 1: assume that ***r*** > R.
   ii. Case 2: assume that ***r*** < R.—the charge is ***inside*** the cylinder.

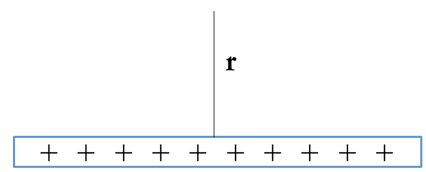
III. Planar Symmetry

Use Gauss's Law to calculate the following. Diagram required for *each* scenario.

a. The (magnitude of the) Electric Field at some distance ***r*** from surface of a large charged plate made of conducting material. A "plate" just means a wide flat area—like a plane. It does not matter whether it is a flat rectangle, a flat circle, a flat oval, etc. It is just big and flat, so big that we don't care about its edges. Since it is flat, we will find the field in terms of ***area*** charge density $\sigma = \frac{q}{A}$.



(Tilted View)



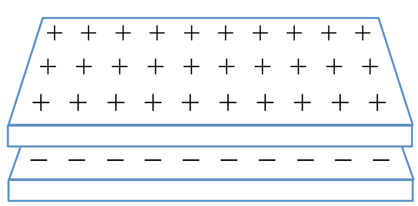
(Side View)

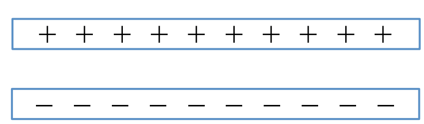
Hint 1: Choose a Gaussian surface that can go straight through the plate like a cookie-cutter. There will be no flux through the sides of your Gaussian shape. All the flux will be through the top & bottom.

Hint 2: Consider the flux through ***both*** top ***and*** bottom.

Hint 3: This case is handled in the textbook, so if you get stuck, look there.

b. Two such charged plates are now placed one above the other with a small space in between (as shown below). The upper plate has positive area charge density σ. The lower plate has negative area charge density $-\sigma$. Find:
   i. The (magnitude of the) electric field at a point in between the two plates;
   ii. The (magnitude of the) electric field at a point a distance r above the upper plate or below the lower plate.





Hint: You can use the same shape as you did for III.a. First imagine it cutting through the top plate, then cutting through the bottom plate. Think hard.

Recall the definition of Electric Potential Difference: $V = \frac{U_{electric}}{q}$

Recall the definition of Potential Energy (in general!):

- ΔU = Work done against the conservative force to move an object through a field.
- ΔU = W that will be done by the conservative force as the object returns to where it started.
- These two definitions are the same: work done against conservative force going away from a point = work conservative force will do returning to that point. ***That's what it means for a force to be conservative***.

Recall the definition of Work: $W = \int_{x_0}^{x} \vec{F} \cdot d\vec{x}$

Recall also the definition of Electric Field: $\vec{E} = \frac{\vec{F}_{electric}}{q}$

Putting these three definitions together, we have:

$$\Delta V = \frac{\Delta U_{electric}}{q} = \frac{\int_{x_0}^{x} \vec{F}_{electric} \cdot d\vec{x}}{q}$$

Since q (the charge on the object) does not change as it moves, we can rewrite the above:

$$\Delta V = \frac{\Delta U_{electric}}{q} = \int_{x_0}^{x} \frac{\vec{F}_{electric}}{q} \cdot d\vec{x} = \int_{x_0}^{x} \vec{E} \cdot d\vec{x}$$

Note we've used x in the above equations to represent position, but you could use some other variable. For example, ***r***.

IV. Consider the single charged plate scenario from problem III.a. above. Again, if you got stuck on this, look in the textbook. Consider two points in space above the plate. Point ***A*** is a distance r_0 from the plate. Point ***B*** is a distance r_1 from the plate.

a. In terms of $\boldsymbol{r_0}$, $\boldsymbol{r_1}$, and $\boldsymbol{\sigma}$, find the difference in electric potential (V) between these two points in space.

b. Imagine that a positive charge of ***q*** is dragged from point ***A*** to point ***B***. In terms of $\boldsymbol{r_0}$, $\boldsymbol{r_1}$, and $\boldsymbol{\sigma}$, what is its change in potential energy (U) for this trip?

c. Imagine that the same charge travels back from ***A*** to ***B***. In terms of $\boldsymbol{r_0}$, $\boldsymbol{r_1}$, and $\boldsymbol{\sigma}$, what is its change in potential energy for this second trip?

d. What is its change in potential energy for the entire round trip?

e. Assume that electric potential (V) is defined to be zero at an infinite distance from the charged plate. Now consider a point some small distance r from the charged plate. In terms of ***r*** and $\boldsymbol{\sigma}$, find the electric potential at this point (i.e. find the difference in electric potential between this point and infinity.)