Space Relations

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I. RELATIVE POSITION VECTORS

A. Relative Positions in 1-D

1. A Pontiac is located *exactly* 60 miles due north of a certain fire hydrant. A Toyota is *also* located *exactly* 60 miles due north of the same fire hydrant. How far apart are the Toyota and the Pontiac? (Note: ***Due*** means "directly" or "exactly" in terms of direction.)
2. At some moment in time, a Pontiac is located 60 miles due north of a gas station. A Toyota is located 30 miles due south of the same gas station. How far and in what direction is the Pontiac from the Toyota? (Draw a picture, please—bird's-eye-view.)
3. At some moment in time, an Pontiac is located exactly 30 miles due north of a Toyota. The Toyota is located exactly 60 miles due south of a gas station.
    - **a.** If the Pontiac is north of the Toyota, which way is the Toyota from the Pontiac?
    - **b.** How far and in what direction is the gas station from the Toyota?
    - **c.** Create a bird's-eye-view diagram of the situation. Use a dot to represent each object. (***Bird's-eye-view*** means looking ***down*** from ***directly above***.) Make sure your diagram matches the given data & is drawn to scale.
    - **d.** How far and in what direction is the Pontiac from the gas station? (I.e., if you walked ***from*** the gas station ***to*** the Pontiac, how far would you walk & what direction?)

B. Relative Positions in 2-D (the easy version)

At some moment in time, a Pontiac is located exactly 30 miles due north of a gas station. The gas station is located exactly 60 miles due east of a Toyota.

1. How far and in what direction is the Toyota from the gas station?
2. How far and in what direction is the gas station from the Pontiac?
3. Create a bird's-eye-view diagram of the situation. Use a dot to represent each object. ***Make sure your diagram matches the given data*** & is drawn to scale.
4. How far and in what direction is the Toyota from the Pontiac? (i.e. if you walked ***straight*** fr the Toyota to the Pontiac, how far would you walk & what direction?)

Ok, so that's relative position. Next comes relative velocity. But here's the trick:
Relative velocities work JUST LIKE relative positions. Literally, same math.
The ONLY difference is that instead of lines and dots that show where things are and how far apart they are, your diagrams will just have arrows that show how things are moving.

II. RELATIVE VELOCITY VECTORS

A. Relative Velocities in 1-D

1. A Pontiac and a Toyota are ***both*** traveling at a constant velocity of 60 miles/hour due north, relative to the interstate highway that they are driving on.

   a. What is the velocity of the Toyota relative to the Pontiac? In other words, imagine you are in the Pontiac. Do you see the Toyota moving further and further north of you? Further and further south of you? How fast?

2. A Pontiac travels at a constant velocity of 60 miles/hour due north, relative to the interstate. A Toyota travels at a constant velocity of 30 miles/hour due south, relative to the interstate.

   a. Draw a diagram of the two cars, as they would appear from a helicopter that is hovering high above the interstate and ***not moving*** relative to the ground. Include ***arrows*** in your diagram to represent the velocity of each car.

   The ***length*** of these arrows should correspond to the ***speed*** of each car. In other words, an arrow for a 20 mph velocity should be about twice as long as an arrow for a 10 mph velocity.

   b. Imagine the two cars are driving straight towards each other (at the given velocities). How fast will they crash into each other.

   c. Now imagine the two cars are driving away from each other (at the same given velocities). How fast will the distance between them increase?

   d. What is the velocity (speed and direction) of the Toyota relative to the Pontiac?

   e. What is the velocity of the Pontiac relative to the Toyota?

   f. What is the velocity of the Pontiac relative to the Pontiac?

3. This one is ***weirder***: A Pontiac travels at a velocity of 30 mi/hr due north, ***relative to a Toyota***. The Toyota travels at a velocity of 60 mi/hr due south, ***relative to a gas station***.

   a. What is the Toyota's velocity relative to the Pontiac?

   b. What is the gas station's velocity relative to the Toyota?

   c. Write out the following vectors using correct notation: V_{PT}, V_{PT}, V_{GT}, V_{TG}

   d. There are two relative velocities missing from the list above. What are they?

   e. Find the missing relative velocities. There are actually two ways to do this:
   Method 1: Vector Addition. Think about how you solved the parallel problem in the relative position section. Use the same method, the same diagram, but with velocity-vector arrows instead of dots and lines.
   Method 2: Vector Subtraction. Create a diagram showing ***the Toyota's frame of reference***. This is a vector diagram how the other two objects are moving relative to the Toyota. It's a weird idea, but if you can dig it, you can solve the problem by working in the Toyota's frame of reference.

f. Draw a vector diagram showing the velocity of the two cars, from the ***gas station's frame of reference.*** The diagram should contain TWO ARROWS: One arrow shows the velocity of the Toyota relative to the gas station. The other arrow shows the velocity of the Pontiac relative to the gas station.

g. Draw a new diagram showing the ***Toyota's frame of reference***: again two arrows: one represents the V of the Pontiac relative to the Toyota. The other represents the V of the gas station relative to the Toyota. (Use GPR 4, Rule #2.)

h. Draw a third diagram showing the ***Pontiac's frame of reference***.

i. Which (if any) of these three perspectives is the ***most correct***? I.e. which one shows how things *really* are? Justify your answer. (2-3 sentences.)

B. Relative Velocities in 2-D (the easy version)

A Pontiac is moving 30 miles/hour due north relative to a gas station. The gas station is moving exactly 60 miles/hour due east relative to a Toyota.

1. How fast and in what direction is the Toyota moving relative to the gas station?
2. How fast and in what direction is the gas station moving relative to the Pontiac?
3. Draw a bird's-eye-view diagram of the two velocities, from the ***gas station's frame of reference.*** The diagram should TWO VECTOR ARROWS: One arrow shows the velocity of the Toyota relative to the gas station. The other arrow shows the velocity of the Pontiac relative to the gas station.
4. What is the velocity (speed and direction) of the Toyota relative to the Pontiac?

III. EXTRA CREDIT

A. Relative Positions in 2-D (the hard version)

A Pontiac is parked 30 miles, in the direction 30 degrees North-East, from a Toyota. The Toyota is parked 60 miles, in the direction 20 degrees South-East, from a certain gas station.

2. Use the following steps to create a bird's-eye-view diagram of this situation:
   a. Draw a dot for the gas station. Label it with a G.
   b. Draw a very light (pencil) east-west line running through the gas station.
   c. Draw a dot for the Toyota and draw an arrow from the gas station to the Toyota. Do your best to estimate a 20° angle south-east from the gas station.
   d. Draw a dot for the Pontiac and draw an arrow from the Toyota to the Pontiac. Do your best to estimate a 30° angle north-east from the Toyota.
3. How far and in what direction is the ***Toyota*** from the ***Pontiac***?
4. How far and in what direction is the ***gas station*** from the ***Toyota***?
5. Draw an arrow from the gas station to the Pontiac.
6. Use everything you know about vectors, components, and trigonometry to find the exact distance, direction, and angle from the gas station to the Pontiac.

B. Relative Velocities in 2-D (the hard version)

A Pontiac travels at 30 miles/hour at 30 degrees North-East, relative to a Toyota. The Toyota travels at 60 miles/hour at 20 degrees South-East, relative to a gas station.

1. What is the velocity of the Toyota relative to the Pontiac?
2. What is the velocity of the gas station relative to the Toyota?
3. Use one of the methods from problem **II.a.3**, step **e** to find the velocity of the Pontiac relative to the gas station.
4. Draw a bird's-eye-view diagram from the gas-station's frame of reference.
5. Draw a bird's-eye-view diagram from the Toyota's frame of reference.
6. Draw a bird's-eye-view diagram from the Pontiac's frame of reference.
7. Which (if any) of these diagrams shows how things are *really* moving (according to physics).