PRACTICE MidTerm 1:

Kinematics in 1 and 2 Dimensions

Physics 203, John Jay College of Criminal Justice

The City University of New York

SOLUTIONS! with Part III

in Preparation For MidTerm Exam 1:

Wednesday, October 17, 2018

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Any/all of THE FOLLOWING RELATIONS underly the material and are hereby

GIVEN (written out for you) below.

I) $d \equiv x - x_0$

II) $\bar{v} \equiv \frac{x-x_0}{t-t_0} = \frac{d}{\Delta t}$

III) $v \equiv \lim\limits_{(t-t_0)\to 0} (\bar{v})$

IV) $\bar{a} \equiv \frac{v-v_0}{t-t_0}$

V) $a \equiv \lim\limits_{(t-t_0)\to 0} (\bar{a})$

VI) ***if and only if*** a_t (for any time t) $= a_0 = \bar{a}$, **then** $\bar{v} = \frac{v+v_0}{2}$

VII) $\vec{v}_{ab} + \vec{v}_{bc} = \vec{v}_{ac}$

VIII) $sin^2\theta + cos^2\theta = 1$

IX) $g \approx 10 \frac{m}{s^2}$ (near surface of Earth)

X) $x - x_0 = \frac{1}{2}at^2 + v_0t$

XI) $x - x_0 = \frac{v^2 - v_0^2}{2a}$

DIRECTIONS/REMINDERS

In ***every problem*** in this exam and in this course, ***you must justify all answers*** in order to receive full credit.

A question might well be answerable ***without any computations; in such cases,*** you must provide some other form of explanation, such as complete sentences of English and/or fully labeled diagrams.

In any event, nearly every solution should include a picture of the facts given by the problem; every variable and constant term relein thvant to the problem should be clarified by a label in the diagram.

PART I ('Warm Ups') (15 pts)

1. A mouse is thrown straight downward off a ***20 meter*** high building.
   The building is located somewhere on the surface of Earth.
   1.8 seconds later, the mouse hits the ground.

   <u>The Question</u>: What was the mouse's initial downward speed (5 pts)?

2. Near the surface of Earth, a helicopter travels upward at a constant velocity of ***60 m/s***.
   At some moment, $\boldsymbol{t = 0}$, a packages drops out of the helicopter.
   The package is neither thrust nor shoved; it is merely released.
   The helicopter continues to move up at the same constant velocity.

   <u>The Question</u>: After ***4 seconds*** have elapsed (from $\boldsymbol{t = 0}$),
   by how many meters are the helicopter and package separated (5 pts)?

3. At $\boldsymbol{t = 0}$, a street hockey stick slaps a puck,
   giving the puck an unknown initial horizontal velocity, $\boldsymbol{v_0}$.
   The street surface underneath the puck is rough,
   so the hockey puck loses speed at every instant of its motion --
   from the very first to the very last.
   The puck slows down at a constant rate:
   Every second, the puck loses the same amount of speed.

   From $\boldsymbol{t = 0}$, the puck takes ***12 seconds*** to come fully to rest.
   During this time, the puck travels ***144 meters***.
   Let the the direction of puck's motion be designated '+', so $\boldsymbol{v_0 > 0}$.

   <u>The Question</u>: What are the magnitude and direction of $\boldsymbol{a}$,
   the constant acceleration for this puck (5 pts)?

Part I.

1.

Given a downward throw for which

$$y = 20\,m,\ t = 1.8\ sec,$$

Find v_0.

Let Coordinate System be y − axis as follows:

$$\uparrow -$$
$$\downarrow +,$$

(i.e.: Down $\equiv$ Positive Direction)

$$y - y_0 = \tfrac{1}{2}at^2 + v_0t$$
$$y - \cancel{y_0}^{\to 0} = \tfrac{1}{2}at^2 + v_0t$$
$$y = \tfrac{1}{2}at^2 + v_0t$$
$$v_0t = y - \tfrac{1}{2}at^2$$
$$v_0 = \frac{y - \frac{1}{2}at^2}{t}$$
$$v_0 = \frac{20\,m - \frac{1}{2}(10\,{}^{m}/_{s^2})(1.8\,s)^2}{1.8\,s}$$
$$v_0 = \frac{20\,m - \frac{1}{2}(10\,{}^{m}/\cancel{_{s^2}})(1.8\,\cancel{s})^2}{1.8\,s}$$
$$v_0 = \frac{20\,m - 5\,(3.24)\,m}{1.8\,s}$$
$$v_0 = \frac{3.8}{1.8}$$

$\boxed{v_0 \approx 2.11\,m/s}$

Part I

2.

Given a package released from a helicopter,

for which $v = v_0 = 60\,m/s$,

and for which $t = 4$ sec from release,

find y.

Let Coordinate System be y − axis as follows:

$$\uparrow +$$
$$\downarrow -,$$

(i.e.: UP $\equiv$ Positive Direction)

HELICOPTER:

$$v = v_0 = \bar{v} = 60\,m/s$$
$$y - y_0 = \tfrac{1}{2}at^2 + v_0t$$
$$y - \cancel{y_0}^{\to 0} = \cancel{\tfrac{1}{2}at^2}^{\to 0} + v_0t$$
$$y = (60\,m/s)(4\,s)$$
$$y_H = 240\,m$$

$$\text{OR } \bar{v} = \frac{y - \cancel{y_0}}{t}$$
$$y = \bar{v}t$$
$$y = (60)(4)$$
$$y_H = 240\,m$$

PACKAGE:

$$y = \tfrac{1}{2}at^2 + v_0t$$
$$y = \tfrac{1}{2}(-10)(4)^2 + 60(4)$$
$$y = -80 + 240$$
$$y_P = 160\,m$$

They both travel UP. So, for DIFFERENCE:

$$\Delta y = y_H - y_P$$
$$\Delta y = 240\,m - 160\,m$$

$\boxed{\Delta y = 80\,m}$

Part I.

3.

Given a puck cominng to rest for which

$$x = 144\,m,\ t = 12\ sec \text{ and } a = a_0$$

Find a.

Let Coordinate System be x − axis as follows:

$$-\longleftarrow \quad \longrightarrow +$$

(i.e.: RIGHT $\equiv$ Positive Direction)

$$\bar{v} = \frac{x - \cancel{x_0}^{\to 0}}{t}$$
$$\bar{v} = \frac{144\,m}{12\,s}$$
$$\bar{v} = 12\,m/s$$

since $a = a_0$ then:

$$\bar{v} = \frac{v_0 + v}{2}$$

but puck comes to rest so:

$$\bar{v} = \frac{v_0 + \cancel{v}^{\to 0}}{2}$$
$$\bar{v} = \frac{v_0}{2}$$
$$v_0 = 2\bar{v}$$
$$v_0 = 24\,m/s$$

and

$$x = \frac{\cancel{v^2}^{\to 0} - v_0^2}{2a}$$
$$a = \frac{-v_0^2}{2x}$$
$$a = \frac{-(24)^2}{2(144)}$$
$$a = \frac{-576}{288}$$

$\boxed{a = -2\,{}^{m}/_{s^2}}$

Part II: CONSTANT ACCELERATION in 1-D (50 pts)

1. A Blood Droplet (25 pts)

Blood has been spattered from a body, but the body has been removed.

One drop of blood has evidently fallen past the entire length of a television set.

The television set is **70.0 cm (+/- .05 cm)*** cm from top to bottom.

Due to the modern miracles of precise programming, DVR and coordinated universal clocks, a criminologist concluded that the drop of blood took **.20 seconds (+/- .005 seconds)*** to fall past the television set.

The drop of blood had evidently fallen from some unknown height above the top of the television set. Wherever it fell from, it was initially stationary.

*The uncertainties are provided because these quantities are meant to be experimentally measured quantities. As would be the case in an actual experiment, you do not need to pay attention to the uncertainties while you do direct analysis. You need only use them when you are asked to do an explicit uncertainty analysis (part g).

Assume that **g ≈ 1000 cm/s^2**. (Whether this value is treated as positive or negative is up to you.)

A. In cm/s, determine the instantaneous velocity, ***v***,

   EITHER at ***t = .10 sec*** from the instant the droplet passes the top of the window
   OR at ***y = 35 cm*** below the top of the window.
   Choose whichever ***one*** velocity you wish, but in order to receive credit, you must specify which instantaneous velocity you have found (4 pts).

B. What was the drop's instantaneous velocity as it passed the bottom of the television (3 pts)?

C. What was the drop's instantaneous acceleration at its peak height (3 pts)?

D. From how high above the television did the drop come (5 pts)?

E. Draw a rough but neat and clear ***v vs. t*** graph of the drop's motion from t = 0 sec to t =**.4** (4/10) seconds.
   Place ***instantaneous velocity*** on the y-axis and ***time*** on the x-axis (5 pts).

F. Refer to your answer for the drop's average velocity past the television screen (part A). An expert witness claims that if certain conditions are true, this victim's blood must have traveled past the screen at an average velocity of 353 cm/sec. Given the measurement digits and uncertainties provided in the fact pattern, explain why your analyzed velocity ***is*** or ***is not*** consistent with this expert witness's prediction. Show all work (5 pts).

rt II.

1. A Blood Droplet

A.

Either at $t = 10$ sec. OR at $y = 35$ cm.

Find v_0. Specify which.

Let Coordinate System be y – axis as follows:

↑ –

↓ +,

(i.e.: DOWN ≡ Positive Direction).

$$\bar{v} = \frac{y - \cancel{y_0}}{t}$$

$$\bar{v} = \frac{70\text{ cm}}{.20\text{ sec}}$$

$$\bar{v} = 350\text{ cm/s}$$

$$a = a_0 = g, \text{ SO:}$$

$$\bar{v} = \frac{v_0 + v}{2}$$

i.e.: $v = \bar{v} = 350$ cm/s MID – WAY through the TIME.

So:

$\boxed{\text{at } t = .10\text{ sec}, v = 350\text{ cm/s}}$

for $y = 35$ cm, solve as follows:

$$y = \frac{v^2 - v_0^2}{2a}$$

$$v^2 = 2ay + v_0^2$$

$$v^2 = 2(1000)(35) + (250)^2$$

$$v^2 = 70000 + 62500$$

$$v^2 = 132500$$

$$v \approx 364\text{ m/s}$$

364 cm/s

but this is much more difficult to do/see.

The real purpose of the question is to focus attention on HALF – TIME.

B.

At bottom of television, find v.

$$\bar{a} = \frac{v - v_0}{t}$$

$$v = at + v_0$$

$$v = (1000)(.1) + 350$$

$\boxed{v_B = 450\text{ m/s}}$

(and, by same reasoning, at top of window, $v = 250$ m/s)

C.

$$a = a_0 = g$$

$\boxed{a = 1000\text{ cm/s}^2}$

D.

$$y = \frac{v^2 - \cancel{v_0^2}^{\to 0}}{2a}$$

$$y = \frac{(450)^2}{2(1000)}$$

$$y = \frac{202500}{2000}$$

$$y = 101.25$$

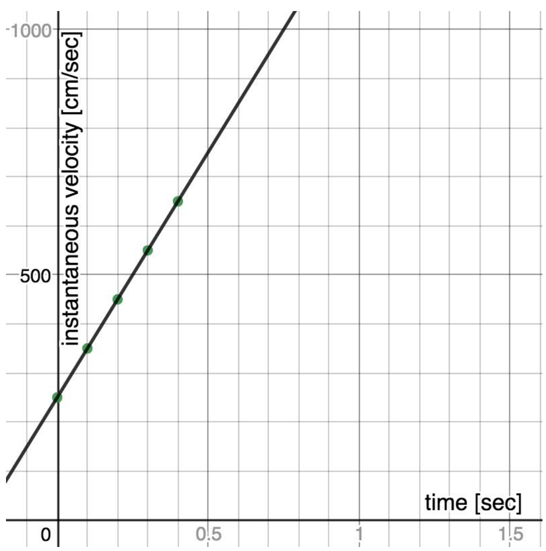
BUT THIS DISPLACEMENT INCLUDES THE WINDOW

(if we used 450 cm/s for final velocity, as above), so:

$$y = 101.25\text{ cm} - 70\text{ cm}$$

$$\boxed{y = 31.25\text{ cm above top of window}}$$

E.



F.

$$\frac{\Delta x}{x} = \frac{.05\text{ cm}}{70\text{ cm}}$$

$$\approx .0007$$

$$\frac{\Delta t}{t} = \frac{.005\text{ s}}{.20\text{ s}}$$

$$\approx .025$$

$$\text{combined uncertainty} \approx .0007 + .025$$

$$\approx .0257$$

$$\approx .0257(350\text{ cm/s})$$

$$\approx 8.99\text{ cm/s}$$

$$\text{so } \bar{v} = 350\text{ cm/s} \pm 8.99\text{ cm/s}$$

The expert witness observed 353 cm/s.

$$350\text{ cm/s} < 353\text{ cm/s} < 359\text{ cm/s}$$

The witness testimony IS within uncertainty range.

$\boxed{\text{The witness testimony IS consistent with analysis.}}$

2. WYLIE COYOTE (25 PTS)

Wily Coyote is waiting at the top of a cliff to drop a heavy weight on the Road Runner, who is running on the road ***below*** the cliff.

At a certain instant, Wily Coyote sees that Road Runner is 40 meters ***due west*** of the bottom of the cliff.

At this very same instant, Road Runner is traveling at an instantaneous velocity of 25 m/s ***due east*** (i.e. ***towards*** the bottom of the cliff).

At this very same instant, Wily Coyote drops the weight (from rest) and starts a stopwatch. Let this one definitive instant be known as ***t = 0 sec***.

At this very very one and the same instant (***t = 0 sec***), the Road Runner sees Wily Coyote and begins ***accelerating west*** at a ***constant*** rate. The magnitude of this rate is 5 m/s^2. Whether you choose to call it + or - depends on other choices that are up to you to make.

A. Assign a coordinate system to the directions relevant to this problem.

Choose 1: Will you call EAST + or - (1 pt)?

Choose 1: Will you call DOWN + or - (1 pt)?

Given the above, draw a quick key (simple compass-like graphic) which explicitly includes all four directions (east, west, up, & down) and assigns a sign (+ or -) to each one (1 pts).

B. For the time interval ***t = 0*** until ***t = 5 sec***, how should the road runner's ***velocity*** and ***speed*** trend -- assuming the Coyote hasn't yet interfered?

CHOOSE ONE of the following four and provide ***one complete sentence*** of explanation (2 pts, 2 pts).

(velocity **decreasing**, speed **decreasing**), (vel. **decreasing**, speed **increasing**)
(vel. **increasing**, speed **decreasing**), (vel. **increasing**, speed **increasing**)

C. For the time interval ***t = 5 sec*** until ***t = 10 sec***, how should the road runner's ***velocity*** and ***speed*** trend -- assuming the Coyote hasn't yet interfered?

CHOOSE ONE of the following four and provide ***one complete sentence*** of explanation (2 pts, 2 pts):

(velocity **decreasing,** speed **decreasing**), (vel. **decreasing**, speed **increasing**)
(vel. **increasing**, speed **decreasing**), (vel. **increasing**, speed **increasing**)

The heavy weight lands exactly on top of the Road Runner.

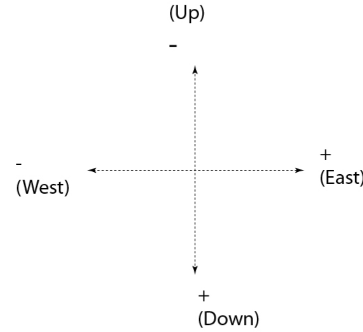
D. ***The Central Question***: **How high was the cliff? (Show all work: 10 pts)**

E. Now suppose the Road Runner must obey the following rule: If/when he ever stops moving, he will never move again. Assume this rule takes precedence over any finding that might conflict with it. In a complete sentence or two, explain how this rule will affect your final answer for the height of the cliff (part *D*, above) (4 pts).

Part II

2.

A.



Any self – consistent set of answers is fine for full credit.

Example:

East +

Down +

Then you must include diagram for full credit (but we did not include one here).

B.

$\boxed{\text{velocity decreasing, speed decreasing:}}$

If East is reckoned positive, then the velocities are positive numbers getting smaller in magnitude,

i.e.: the roadrunner is slowing down in a positive direction while traveling in a positive direction.

C.

$\boxed{\text{velocity decreasing, speed increasing.}}$

This time interval is after velocity has shrunk to zero, so now velocities are increasingly negative numbers:

still decreasing because acceleration is constant, but their magnitudes are increasing,

thereby representing an object speeding up.

D.

First find time of collision; use RoadRunner because we have all the facts:

$$x = \frac{1}{2}at^2 + v_0 t$$

$$40 = \frac{1}{2}(-5)t^2 + 25t$$

$$40 = -2.5t^2 + 25t$$

$$16 = -t^2 + 10t$$

$$t^2 - 10t + 16 = 0$$

$$(t-8)(t-2) = 0$$

$$t = \{2 \text{ sec}, 8 \text{ sec}\}$$

So cliff height:

$$y = \frac{1}{2}at^2 + \cancel{v_0 t}^{=0}$$

$$y = \frac{1}{2}(10)(2)^2$$

$$y = 20 \text{ m}$$

OR

$$y = \frac{1}{2}(10)(8)^2$$

$$y = 320 \text{ m}$$

$\boxed{\text{Height of cliff} = \{20 \text{ m}, 320 \text{ m}\}}$

E.

In this case, roadrunner never turns around and the longer time answer is not possible.

So in this case,

$\boxed{\text{Height of cliff} = 20 \text{ m}}$

PART III : Velocity Addition in 2 - D

Free-Floating (35 pts: 5 pts each paWrt)

A boat travels down a river.

Relative to the water, the boat moves at a constant speed of 12 m/s in a direction of 30º SE.

Relative to the riverbank, the boat moves at a constant speed of 10 m/s in a direction of 45º NW.

A. Relative to ***the boat***, in what direction is ***the water*** moving? PROVIDE POINTS of the COMPASS ***and*** AN ANGLE (3 pts)?

B. Relative to ***the boat***, how **fast** is ***the riverbank*** moving (2 pts)?

C. From ***the water's*** frame of reference, at what velocity is ***the water*** moving (2 pts)?

From the perspective of ***the water***, at what velocity is ***the riverbank*** mroving? (two parts)

D. Find the answer ***graphically***, by drawing vector arrows. Your drawings must be more or less to scale. Each vector must be clearly labeled, along with angle and magnitude. Based on your drawing, write down your best estimate of the speed and direction of the riverbank relative to the water (5 pts).

E. Find an ***exact numerical answer***, using vector components.

You must provide BOTH a magnitude and a 2-dimensional direction (i.e. angle)—in the appropriate units. Show ALL work.

F. For the work in E, provide three relevant and helpful ***triangles*** each composed of ***arrows*** (not just lines) (15 pts total: 3 pts each triangle, 3 pts computation). You may certainly just point to any triangle(s) you already made in the process of doing (E).

G. Create THREE birds-eye-view vector diagrams: one from the boat's frame of reference, one from the water's, and one from the river bank's.

Each diagram should show arrows to represent the velocities of ***each of the objects that is moving*** in that reference frame (6 pts total: 2 pts each).

H. A leaf is floating on the water: it moves at exactly the velocity of the water.

A frog is sitting on a rock in the river: he moves at exactly the velocity of the riverbank. How fast is the leaf moving relative to the boat (3 pts)?

I. At a certain moment, the leaf is right next to the frog. After four seconds, how far is the leaf from the frog, and in what direction (2 pts)?

Part III

A.

$\boxed{\vec{V}_{WB} = 12\ m/s \text{ at } 30^\circ N - W}$

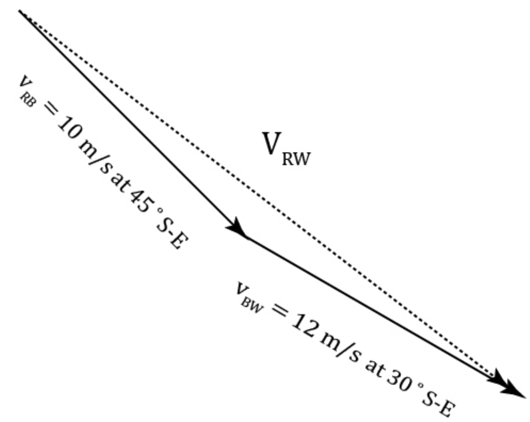
B.

$\boxed{\vec{V}_{RB} = 10\ m/s \text{ at } 45^\circ S - E}$

C.

$\boxed{\vec{V} = 0\ m/s}$

D.

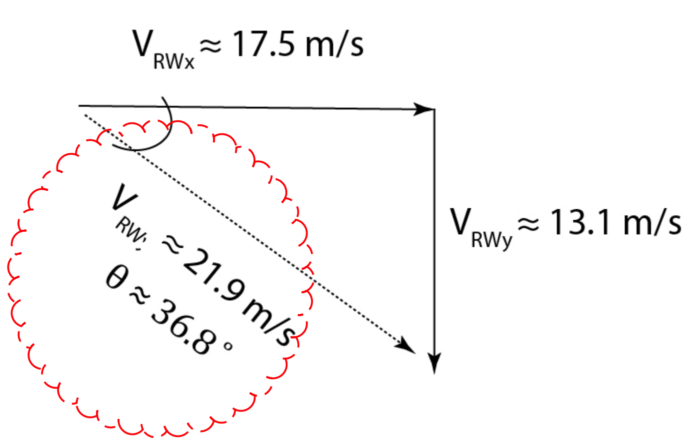


E.

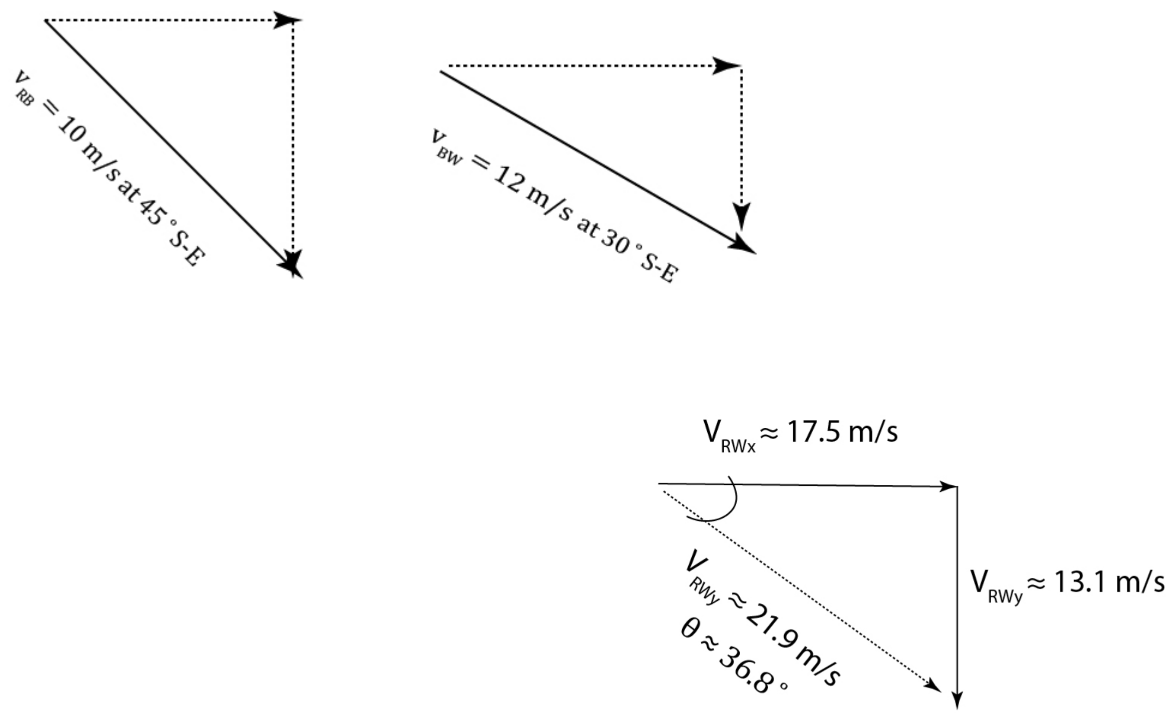
Find $\vec{V}_{RW}$.

$\vec{V}_{RW} = \vec{V}_{RB} + \vec{V}_{BW}$

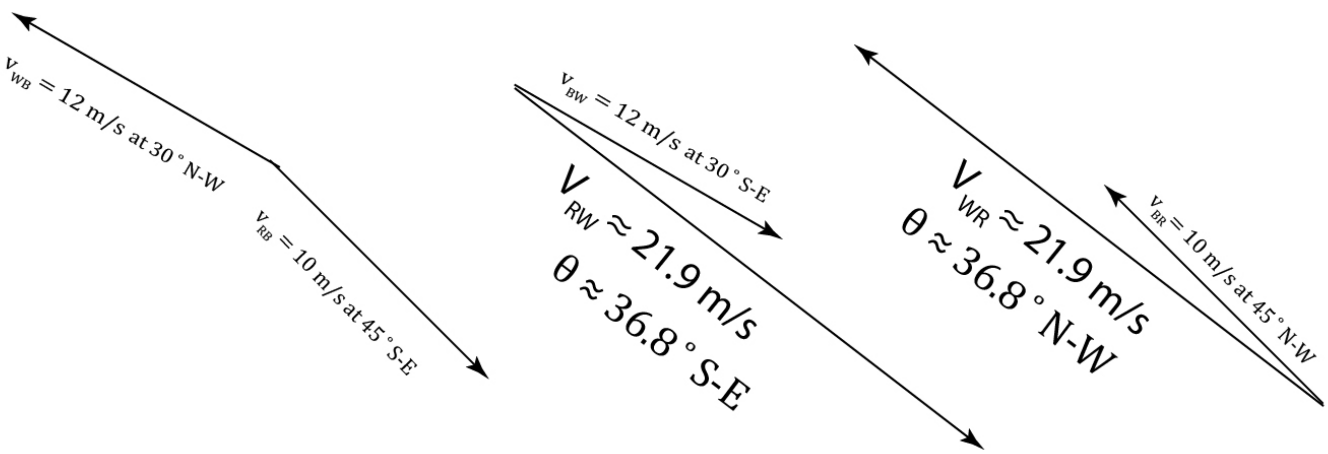
	V_x	V_y
V_{RB}	10 cos 45 ≈ 7.07	-10 sin 45 ≈ -7.07
+ V_{BW}	+ (12 cos 30 ≈ 10.4)	+ (-12 sin 30 ≈ -6)
= V_{RW}	≈ 17.5	≈ -13.1



F.



G.



H) v = 12 m/s

I) x = (21.9)(4)
= approx 88 meters to the northwest.