Midterm 2:

INTERACTION

Physics 203
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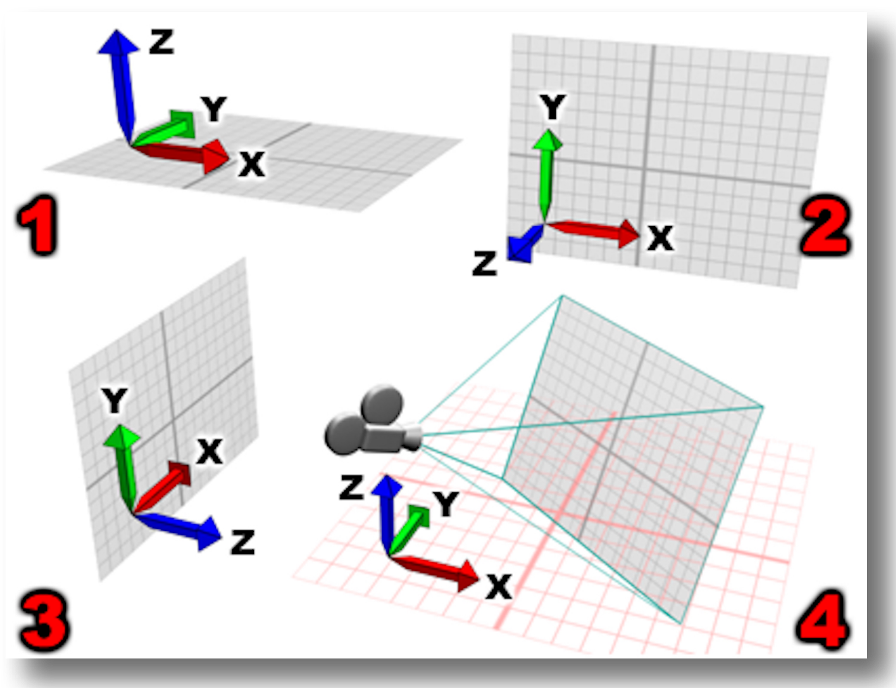
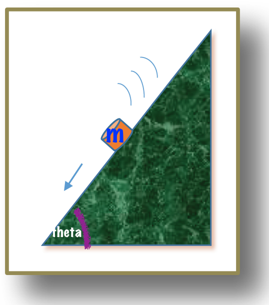


PROBLEM I (35 POINTS):

A small block is ***sliding*** down a ***smooth*** slide at some ***unknown*** but ***constant*** **acceleration.**

The block's mass has been measured; the mass is thereby treated as given and known as $\boldsymbol{m}$**.**
The angle between the slide and the horizontal lab table has been measured; this angle of inclination is given and known as $\boldsymbol{\theta}$.
Assume the acceleration of any object free-falling toward the center of Planet Earth is approx. constant, given and known as: $\boldsymbol{g}$.

YOUR ULTIMATE GOAL: Derive and determine and **a** (the acceleration of $\boldsymbol{m}$) as a *function* of known and fundamental constants ($\boldsymbol{m}$, $\boldsymbol{g}$, $\boldsymbol{\theta}$).



ULTIMATE GOAL: Determine $\boldsymbol{a}$ (the acceleration of $\boldsymbol{m}$) as a function of the constants given above.

A. Provide a '***system schema***' for this situation: For every external object interacting with this mass, use a solid circle and a line segment to convey the interaction (1 pts).

B. Provide a '*pure*' ***free-body-diagram*** of the mass: For every force acting on the mass, use a vector (arrow) in order to convey both the direction in which the force acts and the approximate magnitude of the force compared that of other forces acting on the mass (2 pts).

C. Choose a coordinate system through which to analyze this situation. Make your choice *explicit* by providing a sketch the two axes you will be using as $\boldsymbol{x}$ and $\boldsymbol{y}$. Your axes must be perpendicular to each other yet show a very specific and clear orientation *relative to the standard vertical and horizontal directions that we identify by the pull of gravity toward Earth's center. We take vertical to be parallel with Earth's pull and we will take the page on which you draw to be essentially horizontal.*

   Do you wish your $\boldsymbol{x}$-axis to lie, for example, strictly parallel to the floor and/or to the short edge of this paper? Or do you intend it at, say, 45 degrees compared to the page? Or angled at theta compared to the page? Clockwise or counter-clockwise? Provide unambiguous labels for each of the following: $\boldsymbol{+x}$, $\boldsymbol{-x}$, $\boldsymbol{+y}$, $\boldsymbol{-y}$ (2 pts).

D. 'Zoom in' on every vector which you find to be directed neither parallel nor perpendicular to any of your axes. Resolve it into components (2 pts).

E. Provide a '*component*' ***free-body-diagram*** of the mass.

Every arrow found in this diagram *must* be parallel to one of two coordinate axes (3 pts).

F. Using your diagrams, apply Newton's 2^{nd} Law to one axis: Start by writing the law itself and then, step-by-step, derive an expression for the *normal force* exerted by the surface of the plane onto the surface of the block. This expression will be a function of given and fundamental constants: $\boldsymbol{m}$, $\boldsymbol{g}$ and $\boldsymbol{\theta}$. (Your final expression might wind up leaving out one or more of this list of permitted terms, but NO OTHER term is permitted.) (3 pts).

G. In clear and distinct pairs of English sentences, describe all the *force pairs* which are demanded by *Newton's 3^{rd} Law* in reference to your component diagram (2 pts).

H. Apply Newton's 2^{nd} Law to the other axis. Execute any and all algebra/simplifications necessary in order to arrive at a final expression for $\boldsymbol{a}$, the acceleration of the block, according to the lab frame of reference. Your final answer will be a function of given and fundamental constants: $\boldsymbol{m}$, $\boldsymbol{g}$ and $\boldsymbol{\theta}$. (Your final expression might wind up leaving out one or more of this list of permitted terms, but NO OTHER term is permitted.) (2 pts.)

I. Show that your final expression for acceleration is consistent with your intuitive sense of motion and/or casual observations of typical physical phenomena.

   a. Pick at least three possible numerical values for theta; for each, find and provide the corresponding result for acceleration (2 pts).

   b. In one complete sentence of your own words, explain how the values you provided demonstrate the reasonability of your finding for acceleration (1 pt).

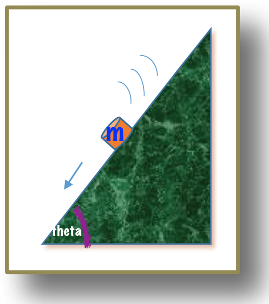
PROBLEM II (20 POINTS):

A small block is ***sliding*** down a ***smooth*** slide at some ***unknown*** but ***constant*** ***acceleration.***

The block's mass has been measured; the mass is thereby treated as given and known as $\boldsymbol{m}$.

The angle between the slide and the horizontal lab table has been measured; this angle of inclination is given and known as $\boldsymbol{\theta}$.

Assume the acceleration of any object free-falling toward the center of Planet Earth is approx. constant, given and known as: $\boldsymbol{g}$.



YOUR ULTIMATE GOAL: Derive and determine and **a** (the acceleration of ***m***) as a *function* of known and fundamental constants (***m***, ***g***, $\boldsymbol{\theta}$).

A. Provide a '***system schema***' for this situation: For every external object interacting with this mass, use a solid circle and a line segment to convey the interaction (1 pts).

B. Provide a '*pure*' ***free-body-diagram*** of the mass: For every force acting on the mass, use a vector (arrow) in order to convey both the direction in which the force acts and the approximate magnitude of the force compared that of other forces acting on the mass (2 pts).

C. Choose a coordinate system through which to analyze this situation. Make your choice *explicit* by providing a sketch the two axes you will be using as ***x*** and ***y***. Your axes must be perpendicular to each other yet show a very specific and clear orientation *relative to the standard vertical and horizontal directions* which we identify by the pull of gravity toward Earth's center.

We take vertical to be parallel with Earth's pull and we will take the page on which you draw to be essentially horizontal.

YOUR CHOICE OF COORDINATE SYSTEM MUST BE DIFFERENT from the one system you used in problem I, above. That is, the x-axis you use here CANNOT be parallel to the x-axis you used in the prior problem.

Provide unambiguous labels for each of the following: ***+x, -x, +y, -y*** (2 pts).

D. ‘Zoom in’ on every vector which you find to be directed neither parallel nor perpendicular to any of your axes. Resolve it into components (2 pts).

E. Provide a ‘*component*’ ***free-body-diagram*** of the mass (2 pts).

Every arrow found in this diagram *must* be parallel to one of two coordinate axes.

F. Using your diagrams, apply Newton’s 2nd Law to one axis: Start by writing the law itself and then, step-by-step, derive an expression for the *normal force* exerted by the surface of the plane onto the surface of the block. This expression will be a function of given and fundamental constants: $\boldsymbol{m}$, $\boldsymbol{g}$ and $\boldsymbol{\theta}$. (Your final expression might wind up leaving out one or more of this list of permitted terms, but NO OTHER term is permitted.) (3 pts.)

G. In clear and distinct pairs of English sentences, describe all the *force pairs* which are demanded by *Newton’s 3rd Law* in reference to your component diagram (3 pts).

H. Apply Newton’s 2nd Law to the other axis. Execute any and all algebra/simplifications necessary in order to arrive at a final expression for $\boldsymbol{a}$, the acceleration of the block, according to the lab frame of reference. Your final answer will be a function of given and fundamental constants: $\boldsymbol{m}$, $\boldsymbol{g}$ and $\boldsymbol{\theta}$. (Your final expression might wind up leaving out one or more of this list of permitted terms, but NO OTHER term is permitted.) (2 pts.)

I. Show that your final expression for acceleration is consistent with your intuitive sense of motion and/or casual observations of typical physical phenomena.

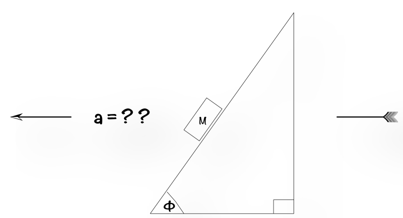
   a. Pick at least three possible numerical values for theta; for each, find and provide the corresponding result for acceleration (2 pts).

   b. In one complete sentence of your own words, explain how the values you provided demonstrate the reasonability of your finding for acceleration (1 pts).

PART III (40 PTS)

A ramp accelerates through a lab. The ***angle*** between the ramp and the (horizontal) floor is a given and known constant – but known simply as ϕ degrees. The ramp accelerates horizontally – along the lab floor. A small block is on the ramp. While this smooth ramp accelerates through the lab, the block remains stuck to it – *stationary relative to the ramp*. The ***mass*** of the block is measured and known, but simply as ***M.*** Similarly, the free-fall acceleration due to gravity is measured and known in this context simply as ***g.***

The coefficient of static friction between the block and the ramp is measured, given and known as μ.



Your ULTIMATE GOALS: You will find values for a_{block}, the rate at which the **block** must *accelerate relative to the lab*, such that the block remains *stationary relative to the ramp.*

CASE I: $\mu = 0$. The ramp is entirely smooth and exerts no friction.

A. Draw a ***System Schema*** for the block in this situation; depict all external objects with which the block is interacting (2 pts).

B. Draw a ***'pure' Free-Body Diagram*** of the block; use one vector to represent each force acting on the block (4 pts).

C. Carefully choose and draw the coordinate system you will find convenient for analyzing the block and determining its acceleration (2 pts).

D. Given your choice, draw the ***component FBD*** of the block; use vector properties to make certain that every arrow in your diagram is parallel to one of the axes of the coordinate system you have chosen (4 pts).

E. Expressed as a function of $\boldsymbol{\phi}$ (and any other given or natural constants, such as ***M*** and ***g***), we wonder: How strong is the '***Normal Force***' exerted by the surface of the plane against the surface of the block?

That is: Find ***N*** in terms of $\boldsymbol{\phi}$, ***g*** **and** ***M*** (4 pts).

F. If the block is to remain perfectly still relative to this smooth plane, then, like the plane, it must be accelerating – through/within the ***lab*** reference frame.

Expressed as a function of $\boldsymbol{\phi}$ (and any other given or natural constants, such as ***M*** and ***g***), we wonder: How great is this acceleration?

That is: Determine $\boldsymbol{a_{(smooth)}}$ in terms of $\boldsymbol{\phi}$, ***g*** **and** ***M*** (4

Again, you might not have to include all of these constants in your answer, but no other non-numerical quantities are permitted in your answer.

** Now, careful measurements are made; we find that ***M*** = **3 kg** and $\boldsymbol{\phi = 55°}$ **

G In Newtons, how hard does the surface of the block push against the surface of the plane (3 pts)?

H. In Newtons, with what strength does the block pull up on planet Earth (3 pts)?

In m/s^2, at what rate does the block accelerate relative to the lab (4 pts)?

***I. For the remaining 10 pts, carefully, clearly and thoroughly derive the full range of values for* a *if* $\boldsymbol{\mu \neq 0}$.**