NII.3: Dynamics & Statics – SOLUTIONS

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I. Friction Warm Ups

A. Moxie, a 4kg calico cat (whose grayish hairs you may occasionally have spied clinging to Prof. Bean's suit jacket), is asleep atop a 1.5 kg laptop computer (which you also may have seen before). The computer sits on a desk. The coefficients of friction between the computer and the desk is given as $\mu_s = 0.8$ and $\mu_k = 0.7$.

   i. Draw a system schema for the whole situation.
   ii. Draw an FBD of Moxie and the laptop as one system.
   iii. Compute the normal force between the laptop and the table.
   iv. Compute the maximum force of static friction between the laptop and the desk?

   At some moment in time, Prof. Bean begins exerting a purely horizontal force of 50 n on the laptop.

   v. Show that Prof. Bean will succeed in moving the laptop.
   vi. Compute the force of kinetic friction on the laptop.
   vii. Compute the acceleration of the laptop across the desk.

   Suddenly, Moxie wakes up, leaps off the laptop, and goes off to do something important in the kitchen. Assume that professor bean continues pushing with the same force as before.

   viii. Compute the new force of kinetic friction on the laptop.
   ix. Compute the new acceleration of the laptop across the desk.

i. [not provided. Should include cat, laptop, desk, and earth (and, later, prof Bean).]

ii. [not provided. Should have just 2 vertical arrows (mg & normal) at the beginning, later add 2 horizontal arrows for Bean's push & friction.]

iii. $\Sigma F_y = ma_y$
$mg - N = 0$ ← treating down as positive; system is not moving up-down, so $a_y = 0$
$N = mg = 55$ n ← m = mass of whole system = 1.5kg + 4kg

iv. $f_s(\max) = \mu_s N = (0.8)(55) = 44$ n

v. Assume system will NOT move, i.e. a=0
$\Sigma F_x = ma_x$
$F_{Bean} - f_s = 0$
$f_s = 50$ ← impossible: $f_s > f_s(\max)$, so our assumption was wrong, the system will move.

vi. $f_k = \mu_k N = (0.7)(55) = 38.5$ n

vii. Use NII in the x direction
$F_{netx} = ma_x$
$F_{Bean} - f_k = (5.5\text{kg})a_x$
$50\text{n} - 38.5\text{n} = (5.5\text{kg})a$ ← treating Bean's push as pos, therefore friction as neg.
$a = 11.5\text{n}/5.5\text{kg} = 2.09\ \text{m/s}^2$

viii. N has changed: Fnet=may → $mg + N = 0$ → $N = -mg = -15$ n
$|f_k| = \mu_k N = (0.7)(15) = 10.5$ n

ix. $F_{netx} = ma_x$ → $N_B + f_k = (1.5\text{kg})a_x$ → $50\text{n} - 10.5\text{n} = (1.5\text{kg})a$ → $a = 39.5/1.5 = 26.33\ \text{m/s}^2$

B. A 22kg sack of apples is lying on the ground. The coefficients of friction between the ground and the bag are mu_s = 0.6 and mu_k = 0.4. An old lady grabs the bag of apples and begins pulling it diagonally upwards with a force of 100N, at an angle 60 degrees above the horizontal.

i. Draw a regular diagram (drawing) and a System Schema for the situation.
ii. Draw a pure FBD and a component FBD of the sack of apples.
iii. Compute the normal force between the sack and the ground.
Hint: set up an NII equation for the y components.
iv. Compute the maximum force of static friction on the bag of apples.
v. Show that the woman will ***not*** succeed in moving the bag.
Hint: set up an NII equation for the y components.
vi. The woman now pulls with a force of 140 N.
vii. Compute the normal force between the sack and the ground.
viii. Show that the woman ***will*** succeed in moving the bag.
ix. Compute the force of kinetic friction.
x. Compute the acceleration of the bag.

i. not provided: four objects: sack, lady, ground, earth.

ii. not provided: Four forces on sack. Lady's force is at a diagonal—split into horizontal (100cos(60) = 50 n) & vertical (100sin(60) = 86.6 n) components.

iii. T_y (***upward*** tension fr lady) = 86.6 N. That's less than mg (220N), so stack stays on ground

$\Sigma F_y = ma_y$

$mg - T_y - N = 0$ ← treating down as positive; stack stays on ground, so $a_y = 0$

$220 - 86.6 - N = 0$ → $N = 133.4$ n

iv. $f_s(max) = \mu_s N = (0.6)(133.4) = 80.04$ n

v. Assume bag will NOT move, i.e. a = 0

$\Sigma F_x = ma_x$

$T_x - f_s = 0$

$f_s = 50n$ ← $f_s < f_s(max)$. Friction IS strong enough to keep the bag from moving.

vi. Not a question. Oops.

vii. $T_y = 140\sin(60) = 121.24$ n. That's still less than mg (220N), so stack stays on ground.

$\Sigma F_y = ma_y$

$mg - T_y - N = 0$ ← because stack stays on ground, $a_y = 0$

$220 - 121.24 - N = 0$ ← treating down as positive

$N = 98.76$ n

viii. Assume the sack does not move:

$f_s(max) = \mu_s N = (0.6)(98.6) = 59.16$ n

$T_x = 140\cos(60) = 70$ n.

$\Sigma F_x = ma_x$ → $T_x - f_s = 0$ → $T_x = f_s = 70n$.

$f_s < f_s(max)$. Friction IS strong enough to keep the bag from moving.

ix. $|f_k| = \mu_k |N| = (0.4)(98.6) = 39.44$ n

x. a is in the x direction, so we have: $\Sigma F_x = ma_x$

$T_{L,x} - f_k = (18)a_x$ ← treating friction as neg & direction of motion as positive.

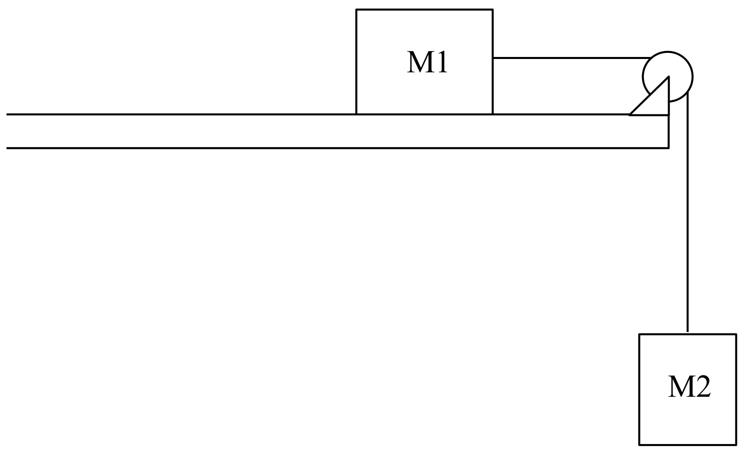
$70 - 39.44 = (18)a$

$a = 30.56/18 = 1.70\ m/s^2$

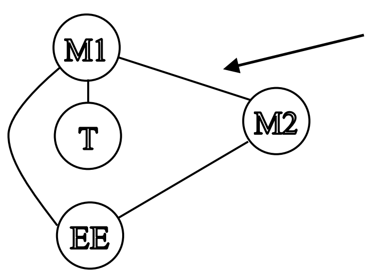
II. Double Block with String

M1 sits on a table and is attached by a string to M2, which hangs off the edge of the table, as shown. The string runs over a pulley wheel at the edge of the table. The string is massless and the pulley wheel is massless & has zero friction at its axel—in other words, it changes the direction of the string & thus of the force of tension, without absorbing any of that force. In short, the force of tension on M1 is equal & opposite to the force of tension on M2.

The coefficients of static and kinetic friction between M1 and the table are given as $\mu_s = 0.5$ and $\mu_k = 0.7$.



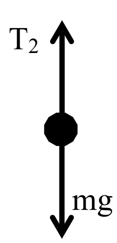
A. Draw a system schema for this situation. (You need not include the string or the pulley wheel in your SS. Connect M1 directly to M2.)

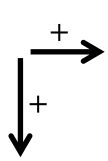


Note: this line represents the tension force that M1 exerts on M2 and the tension force that M2 exerts on M1. All lines in a System Schema represent Newton's 3rd Law pairs, but unlike most system-schema lines this force is bent because of the pulley, so the force on M1 pulls ***to the right***, but the force on M2 pulls ***upwards***.

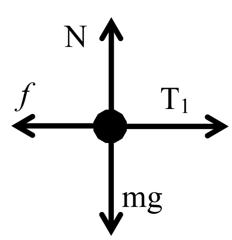
Assume that M1 has a mass of 10kg and M2 has a mass of 8kg.

B. Draw an FBD of M2.





D. Draw an FBD of M1



Notice that if the system were to accelerate (due to gravity on M2), M1 would go ***right*** and M2 would go ***down***. We made both right and down positive, so that the ***a*** would be positive on both blocks.

C. Create an NII equation for M2. You will have two unknowns in your equation.

All forces are in the y-direction.

$Fnet_y = ma_y$

$mg - T_2 = 8a_y$ ← TM2 is going to turn out to be negative, but we are still ADDING up all the forces.

$80 - T_2 = 8a_y$ ← We don't yet know a_y or T, so we leave these as unknown.

D. Draw an FBD of M1. [Answer on preceding page]
E. Create an NII equation for M1 on the y-axis.
F. Compute the normal force on M1.
G. Compute the force of friction on M1.

$Fnet_y = ma_y$

$mg - N = ma_y$

$100 - N = 0$ ← $a_y = 0$, because M1 cannot be moving up or down.

$N = 100$

Ooops! These instructions are ambiguous: are we computing the force of ***static*** friction or ***kinetic*** friction? That was probably a mistake on the part of whoever wrote this problem (who may or may not have been me), but looking ahead I see that I'm going to have to compute the ***acceleration*** of the system, which I guess means it ***is*** moving, so it must be ***kinetic*** friction.

$f_k = \mu_k N$

$f_k = 0.7(100) = 70\ n$

H. Create an NII equation for M1 on the x-axis. You will have two unknowns in your equation.

$\Sigma F_x = ma_x$

$-f_k + T_1 = 10a_x$ ← friction is left so it's neg. a_x & T_1 are right, so they're positive.

$-70 + T_1 = 10a_x$

I. Your NII equations for M1 and for M2 have the same two unknowns. Solve them simultaneously (as a system of two equations) to find the acceleration of the system and the tension on the string.

$-70 + T_1 = 10a_x$ $\quad\quad$ $80 - T_2 = 8a_y$ ← T_2 is up, so negative; a_y is down so positive.

Wait a sec, YaverBean! You said these two equations had the same two unknowns, but hey don't! I see a T_1 and a T_2, and an a_x and an a_y! What gives?

But look: if the blocks slide, they're going to slide at the same rate (since the string isn't stretchy), so M1's a_x right = M2's a_y down. And it just so happens that we defined right and down both as positive, so $a_x = a_y = a$. We also know that T right on M1 = T up on M2, so $T_1 = T_2 = T$.

So, $\quad -70 - T = 10a \quad$ and $\quad 80 + T = 8a$

Add the equations:

$$\begin{aligned} -70 - T &= 10a \\ \underline{80 + T} &= \underline{8a} \\ 10 &= 18a \\ a = 10/18 &= 0.556\ m/s^2 \end{aligned}$$

Note: This is ***one*** way to solve this pair of equations. You could also solve for T in both equations and then set them equal—or use substitution. Lots of options.

Now assume that M1 has a mass of 12kg but M2 has an unknown mass.

J. Using your FBD of M1 & M2 from step B, compute the minimum mass that M2 must have in order for the system to accelerate.

Hint: Look at the steps you performed in A-I above. The same kind of thinking, with different knowns & unknowns, can help you solve question J.

If we are interested in the minimum mass needed to make something start accelerating, we are asking about WHETHER a thing will move, so that's a STATIC FRICTION question.

For M1, we have:
$\Sigma F_x = ma_x$
$-f_s + T_1 = ma_x$
f_s pulls left, T_1 pulls right. System will move if T_1 is stronger than f_s

In other words, the system will accelerate if the force of tension pulling on M1 is strong enough to beat the static friction force between M1 and the table—in other words, if T is stronger than f_s(max).

The system will move iff: $\mathbf{T_{M1} > f_{s,max}}$
$f_{s,max} = \mu_s N$ ← i.e. f_s can pull with a strength of ***up to*** but not more than $\mu_s N$
So we have to find N. Use ΣF_y.
$\Sigma F_y = m_1 a_y$
$m_1 g - N = m_1 a_y$
$120 - N = 0$ ← $a_y = 0$, because M1 cannot be moving up or down.
$N = 120$
$f_{s,max} = \mu_s N = 0.5(120) = 60\ n$ ← This is the maximum force that f_s can pull with.
System will move if $T_1 > 60\ n$

Assume the system ***starts*** at rest. Will it begin to move?
If system is at rest, then $a_{M1x} = a_{M1y} = 0$

<u>M2</u>
$\Sigma F_y = ma_y$
$m_2 g - T_2 = 0$
$m_2 g = T_2$
But $T_1 = T_2$ (by NIII: M1 pulls on M2 w/ the same force that M2 pulls on M1.)
So, $T_1 = m_2 g$, so if $m_2 g > f_s$(max), then the system will move.
So, $m_2 > 60/g = 6$ kg

Mass of M2 must be greater than 6 kg for the system to begin to move.