2. The Inclined Plane is Moving!

A child seated in a car places a quarter on a clipboard.
The coefficient of static friction between the clipboard and the quarter is 0.1.
The child tilts the clipboard forward (i.e. front end down, back end up).

The car is accelerating forward at a constant rate of 2 m/s^2.

Ultimate GOAL of this problem:
Find the *range* of angles (measured from the horizontal) at which the clipboard can be tilted without any sliding of the quarter;
put another way, determine all the tilt-angles for which the quarter remains stationary relative to the clipboard.

* GUIDE! *

- Of course, you should right away draw an informal but well-labeled PICTURE of the situation described by the Question. You need to start with a picture of the problem whether or not you are explicitly reminded – and you really should do so before beginning to pursue any kind of solution. What is going on in this scenario? What features and quantities are essential? Include and label them.

- The problem announces the ultimate goal – the answer you'll be directly pursuing toward the end of all the problem parts. Problems don't often describe their ultimate goal in advance, but they generally do have one.

  Whether you peek ahead or predict from practice, it's helpful to be softly aware of a problem's destination while you solve the earlier parts. These earlier parts often set up pieces you need to put together. Each part, if considered in this light, often provides a powerful clue or two for how to solve another part. Pay attention to the point values. If the point value for a certain part is quite low, then the part might well be just as quick and straightforward to solve for as you hope; it's there to direct your attention to a piece of information that you're going to need for the ultimate goal.

- The problem asks for a RANGE of values as an answer.

  - That might seem out of nowhere, but it is not being asked just to be obnoxious. At some point, you're going to have to consider why more than one numerical answer (for angle) is possible and relevant to this particular situation.

  - It's true: more than one angle will accomplish the stated goal. The size of a suitable angle, that is, can VARY. Some quantity on which the answer depends must vary. Can you think of any physical quantity relevant to the problem that can take on more than one value – one which demonstrates the freedom to 'adjust' to the needs of a situation?

  - Many, many values can be included in a typical range. But if you solve directly for just a couple of key values, you can use them in order to capture and express the entire range of all possible values. Which 'couple' of key values do you need?

- Remember: the ultimate goal specifically describes the quarter's required velocity as 'relative to' the clipboard. As opposed to what? If the quarter is moving in a certain way relative to the clipboard, then what is it doing in any other possibly relevant reference frame? Can you make this distinction clear in your picture?

A. If we are looking at the minimum angle, what direction will friction have to point to keep the quarter in place?

- How do we determine the direction of friction IN GENERAL? That is, the friction exerted by surface B onto surface A must . . . ?

- How do we determine the direction of forces in general? We ask ourselves: If the force WERE NOT acting at all, in which direction would the system (or 'object of focus') tend to accelerate? If the force is, in fact, acting and the object is, in fact, NOT accelerating in that fashion, then what does this say about the direction in which the force is acting?

- So . . . , at a very low angle (like the minimum), in the absence of friction, what would the quarter tend to do? Therefore, at such an angle n which direction must friction point?

B. If we are looking at the maximum angle, what direction will friction have to point to keep the quarter in place?

C. If the angle goes below the minimum angle, what will happen to static friction and what will happen to the quarter?

- This is a qualitative question – not demanding any particular computations or numbers – but it does contain the word 'and': Make certain to address *both* parts of the question.

D. If the angle goes above the maximum angle, what will happen to static friction and what will happen to the quarter?

- (Many copies of the original question from the original practice exam say "minimum angle". That is simply a typo—it should read "maximum angle".

- This question is extremely similar to the question before it, but it would hardly be worth asking both on an exam unless the expected answers differed from each other in some respect. This is a clue: Your answer to (C), above, might have been correct, but if it does not lead to some kind of distinct answer here in (D), then it might not have been *specific* enough.

E. Draw a pictorial diagram of the situation, including all known and unknown quantities.

- If you have followed good practice and already done this, do not panic nor second guess yourself. You need not recopy. Just write a clear instruction telling the reader how to find the relevant work, such as "See page 3, top".

F. Draw a system schema for the situation.

Find the minimum angle:

- Note that this instruction does not have a letter before it (or a point value) and is 'outdented'. It is indicating a goal toward which the next few parts are heading. You aren't expected to find the numerical answer until you get to the end of such parts.

G. Draw a pure FBD of the quarter for this scenario (minimum angle).

- What is the object of focus?

- Be very careful when you draw this pure FBD. Note that drawing a pure FBD is one of many instructions we follow always – as part of a standard procedure. A standard procedure remains standard no matter how non-standard or challenging a circumstance might seem. That is, we don't throw away our life raft when waters start to look choppy. That's why we have the life raft in the first place. So draw your FBD the way you always do – from the kind of perspective or reference frame that you (and/or Galileo and/or Newton) always assume.

H. Choose a coordinate system in which the x-axis is parallel to the direction of acceleration (assuming that the quarter does not slide).

- "The direction of acceleration" necessarily means 'the direction in which the object of focus accelerates'. Think carefully about this – particularly in relation to the second hint for (G), above.

I. Draw a component FBD of the quarter.

- Again, be very careful here. You might have drawn component FBD's for inclined planes before, but do not simply rely on memory. You must break up forces into components that lie along coordinate axes. Were the components in your memory necessarily drawn for the coordinate system you are currently using? Why or why not?

- Write NII equations for the quarter on both axes.

- As always: Apply the law independently to each of two axes. And, as always, don't forget to consider: With which axis would it be wise to start?

J. Express fs(max) in terms of N.

K. That is, wherever you see the term fs, substitute the traditional expression for fs(max). Ask yourself: Why is it appropriate to assume that static friction is acting at its maximum strength?

L. Solve for N in terms of other variables in both equations.

- That is, re-arrange each equation so that N is isolated on one side of an equation.

- Why? Is anybody asking you N? Is that what you ultimately care about? Why would you rearrange both equations to solve for it? Hint: Solving both equations for N allows you to do what with both equations?

Hint 1: The quarter is not sliding.
Hint 2: Do plug in 10 for g.

M. Solve to find ***θ(min)***.

Find the maximum angle.

N. Solve to find ***θ(max)***, using the same steps as you used to find ***θ(min)***.

So, you have now solved for precisely two angles. Have you reached the 'ultimate goal' of this problem? Have you expressed your last answer as a direct response to the ultimate question?

Circle your final answer. Make it something worth circling . . .