Work Done by Changing Forces

PHYSICS 203, PROFS. BEAN & MARTENS YAVERBAUM
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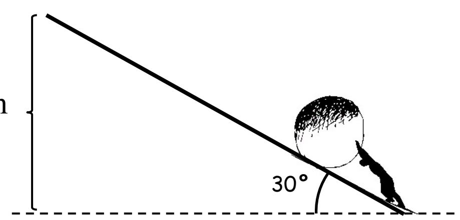
Complete the textbook readings: 7-3, 7-4, 7-5, 8-1, 8-2

Recall the definition of work ***when forces are constant***: $\vec{F} \cdot \vec{d}$
(refer to textbook section 3-3, "The Scalar Product")

Recall the definition of work ***when forces are NOT constant***: $\int_{x_i}^{x_f} F_x dx$
(where x_i is the initial location, x_f is the final location, and F_x is a force acting parallel to the displacement: neg if same direction, pos if opposite.)

I. BOULDER REVISITED

Sisyphus is rolling is rolling a 100 kg boulder up another ramp. This ramp as a height of 12 m and an angle of 30 degrees, and friction exerts a force of only 50 Newtons.

12 m


A. Calculate the work done by gravity on the boulder as Sisyphus pushes it all the way up the inclined plane. (Is this work negative or positive?)
B. Calculate the work done by friction on the boulder as Sisyphus pushes it all the way up the inclined plane. (Is this work negative or positive?)
C. Sisyphus starts out pushing the boulder with a force of 990 N, but he gradually gets weaker as he climbs. For each meter he travels, he loses 30 N of force. So, after one meter, he's only pushing with 960 N; after two meters, he's pushing with 930; and so on.
   i. Write the force of Sisyphus's push as a function of his position, x.
   ii. Calculate work done by Sisyphus as he pushes the boulder all the way up the ramp.
D. Assuming that the boulder starts out from rest, how fast is it moving by the time it reaches the top of the ramp?

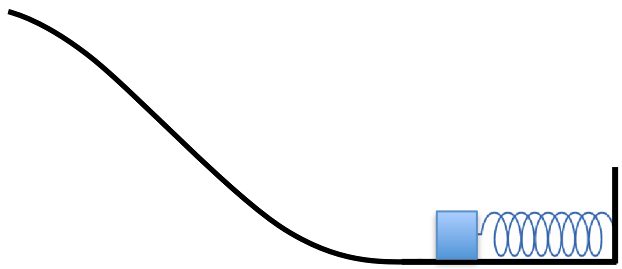
Hints for problem II. C. i.
Hint 1: Define your x-axis to be the ramp, so that x=1 means 1 meter up the ramp, and so on.
Hint 2: Your function will look like $F_{sys} = A - Bx$, where A & B are numbers that you fill in.

Hints for problems II. C. ii.
Hint 1: Is the force in the direction of the displacement? Is the force constant?
Hint 2: Since the force [is]/[is not] constant, which definition of work should you use? The dot-product or the integral?

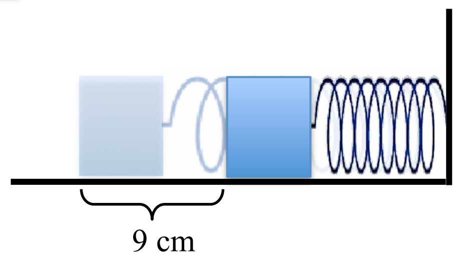
Hint 3: Remember, you already found F as a function of x, so you can plug this function into the definition of work. You will be integrating with respect to x, so that function is already in the form you need it to be in order to integrate.

II. Bouncy Bouncy Bouncy Bouncy…

A perfect, massless spring with spring constant k = 30,000 N/m is affixed to a wall at the base of a ramp. A block (mass = 3kg) is touching the end of the spring that is away from the wall. The block begins at rest, and the spring begins at equilibrium (i.e. neither stretched nor compressed).



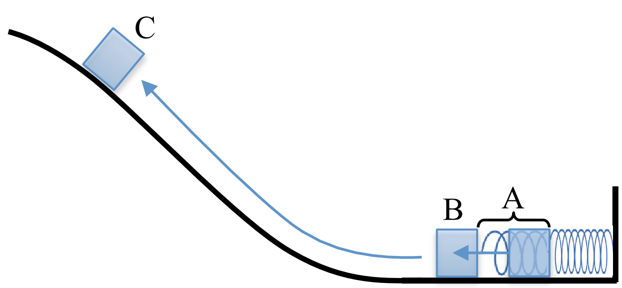
A mysterious (yet somehow familiar seeming) stranger comes along and presses the block towards the wall, compressing the spring a distance of 9cm. Then she releases it.

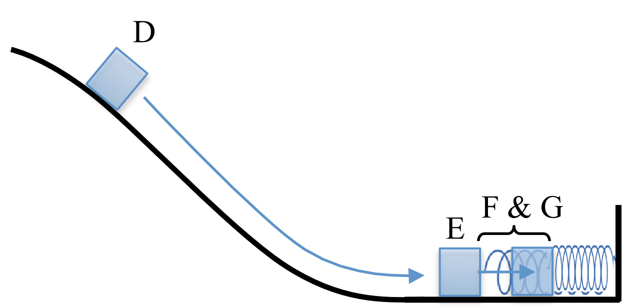


A. Calculate the work done by the spring as it returns to equilibrium.
B. Calculate the kinetic energy of the block the moment the spring reaches equilibrium.

Assume that friction is negligible.

C. Calculate the *height* (d_y) that the block will travel up the ramp before coming to rest.
D. After coming to rest on the frictionless ramp, the block will of course begin to slide back down. Calculate the work done by gravity as the block slides back down.
E. Calculate the kinetic energy of the block when it gets back to the bottom of the ramp.
F. Naturally, the block will continue sliding (on the frictionless track) until it crashes back into the spring. Calculate the work the spring will have to do in order to stop the block.
G. Calculate how far the spring will be *compressed* as it stops the block.



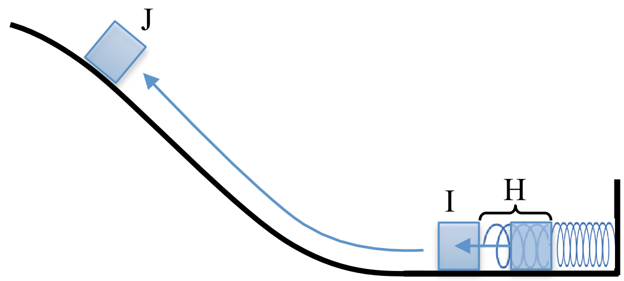


Letters in diagrams refer to question parts

(Problem II, continued)

So, the block will crash into the spring, compressing it a distance of 9 cm (yep, that's what you should have gotten for G), before coming to rest. Then, of course, the spring will push the block back out (since, after all, it's a spring, and it *always wants to return to equilibrium*.)

H. Calculate the work the spring will do on the block as it returns to equilibrium.
I. Calculate the KE of the block the moment the spring reaches equilibrium.
J. Calculate the height the block will travel up the ramp before coming to rest.
K. How long will this process continue? Describe what will happen over time.



What you have discovered is that ***a spring force*** is a type of ***conservative force***.

L. Write down the definition of a conservative force (from lecture) & explain how the work you did in parts A, B, E, F, G, H, and I shows that a spring force is a conservative force.

We will now introduce a *****new concept**— POTENTIAL ENERGY***.

Notice that, when the block is at the top of the ramp, gravity has done a bunch of ***negative work*** in order to bring the block to rest. It's now ready to do a bunch of ***positive work*** as the block travels back down.

Similarly, when the spring is compressed 9 cm, the spring has done a bunch of ***negative work*** to stop the block and is now ready to do a bunch of ***positive work*** as it returns to equilibrium.

In each case,
the negative work on the trip against the force = the positive work on the way back,
because both spring and gravity forces are ***conservative***.

Any time a conservative force does some negative work on an object, it then has the ability (the ***potential***) to do the same amount of positive work as the object returns to its starting point. This positive work that the conservative force is ready to do is called ***POTENTIAL ENERGY***, and it is represented by a capital letter ***U***.

So, ***potential energy*** is ***created*** when a conservative force does ***negative work***, and potentially energy is ***used up*** when a conservative force does ***positive work***. So,

$\Delta U \equiv$ the ***negative*** of the work that a conservative force ***has done***; in other words,
$\Delta U \equiv$ the ***positive*** of the work that the conservative force ***is ready to do***;

(Notice that what we've actually defined is ***change in*** Potential Energy. We'll discuss this more in class, but for now you can just ignore the deltas in the above definitions.)

For the problem we were dealing with just now, with the block and the spring, we'll say that potential energy is zero when the block is ***at the bottom of the ramp*** (i.e. gravitational potential energy = 0) and the spring is ***not compressed*** (i.e. spring potential energy = 0). So, in the positions labeled B, E, and I in the diagrams above, we'll say the block has zero potential energy.

M. As the block travels up the ramp, what is happening to its kinetic? What is happening to its potential energy?
N. What is the block's gravitational potential energy when it reaches the top of the ramp?
O. What is the block's gravitational potential energy when it has slid back down the ramp to a height of ***1 meter above the bottom of the ramp***.
P. What is the block's potential energy when it gets all the way back to the bottom but has not yet impacted with the spring?
Q. What is the block's spring potential energy when it has compressed the spring a distance of 3 cm?
R. What is the block's spring potential energy when it has compressed the spring a distance of 9 cm?

In parts A-K above, you made a bunch of predictions about the block, based on the assumption of no friction. When you actually perform the experiment, however, you discover that the block travels up the ramp to a *vertical height* of only 3 meters before coming to rest, due to the presence of ***a non-conservative force***: friction.

S. Based on your answer to (B) above, what is the net work that must have been done on the block as it slides up the ramp, in order to bring it to rest (at a height of 3 meters)? In other words, how much net work must have been done on the block from the moment the spring released it until the moment it comes to rest at the top of the ramp.
T. What TWO forces did this work?
U. Calculate the work done by each of those forces.
   Hint 1: Find gravitational potential energy first. That you can get directly.
   Hint 2: You don't know the force of friction, so you will have to find the work it did by working backwards, using the Work-Kinetic Energy theorem.
V. In order to reach a height of 3 m, the block had to travel a distance of 8 m along the track. Assuming that the force of friction was constant throughout that trip, calculate the constant force of friction during the trip.
W. As the block slides back down the ramp to the spring, how much work would you expect friction to do? More, less, or the same as on the way up? Negative or positive?