Space Relation - Part 2

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Recall the following terminology:

$\vec{V}_{AB} \equiv$ the <u>**velocity**</u> ***of*** object ***A***, ***relative to*** (i.e. ***from the perspective of***) object ***B***

Similarly,

$\vec{X}_{AB} \equiv$ the <u>**position**</u> ***of*** object ***A***, ***relative to*** (i.e. ***from the perspective of***) object ***B***

In Vector Addition & Spatial Relations, Part 1.I, we observed three rules of ***relative position***:

Rule #1: $\vec{X}_{AA} = 0$: every object is always 0 distance away from itself.

Rule #2: $\vec{X}_{AB} = -\vec{X}_{AB}$: e.g., if A is 10 m ***north*** of B, then B is 10 m ***south*** of A.

Rule #3: $\vec{X}_{AB} + \vec{X}_{BC} = \vec{X}_{AC}$: if you go from A to B and then from B to C, the result is the same as going straight from A to C.

It turns out (as we observed in Vector Addition & Spatial Relations, Part 1.II), the exact same rules apply to ***relative velocity***:

Rule #1: $\vec{V}_{AA} = 0$ (See Space Relations Part 1, problem II.A)

Rule #2: $\vec{V}_{AB} = -\vec{V}_{AB}$ (See Space Relations Part 1, problem II.C)

Rule #3: $\vec{V}_{AB} + \vec{V}_{BC} = \vec{V}_{AC}$ (See Space Relations Part 1, problems II.D & II.E)

Rule #1 is straightforward. It says: *the velocity of any object relative to itself is zero*.

To understand **Rule #2**, you have to know how negative vectors work: if V is a vector, -V is a vector with the ***same magnitude*** & ***angle*** as V but the ***opposite direction***: e.g. 30 mph at 40° NE becomes 30 mph at 40° SW. So this rule says that object A sees object B moving at the same speed and in the opposite direction from how object B sees object A moving.

The classic form of **Rule #3** (from Galileo himself) goes like this:

Your V relative to the sun = your V relative to Earth + Earth's V relative to the sun.

The key to this rule is understanding that ***adding vectors*** does ***not*** mean adding their magnitudes. To add two vectors, draw them tip to tail. The sum of the two vectors will be the vector that goes straight from the tail of the first to the tip of the second.

From now on (in this course), these three rules for relative velocity will be known as

Galileo's Principle of Relativity, Form 4

You may notice that GPR 4 is a bit different from GPR 1, 2, and 3. The first three forms are statements about ***the laws of physics***. Form 4 is a ***mathematical description*** of relative velocity.

In the next few problems, you will apply GPR 4 to calculate some relative velocities. You may find that you can figure the first couple out intuitively, but as they get harder, GPR 4 will help.

I. RELATIVE VELOCITY VECTORS – In Depth

A. Relative Velocities in 1-D

i. A Pontiac and a Toyota are ***both*** traveling at a constant velocity of 60 miles/hour due north, relative to the interstate highway that they are driving on.

a. Draw a diagram of the two cars, as they would appear from a helicopter that is hovering high above the interstate, and ***not moving*** relative to the ground. Include ***arrows*** in your diagram to represent the velocity of each car.

b. What is the velocity of the Toyota relative to the Pontiac? In other words, imagine you are in the Pontiac. Do you see the Toyota moving further and further north of you? Further and further south of you? How fast?

ii. A Pontiac travels at a constant velocity of 60 miles/hour due north, relative to the interstate. A Toyota travels at a constant velocity of 30 miles/hour due south, relative to the interstate.

a. Draw a diagram of the two cars, as they would appear from a helicopter that is hovering high above the interstate, and ***not moving*** relative to the ground. Include ***arrows*** in your diagram to represent the velocity of each car.

The ***length*** of these arrows should correspond to the ***speed*** of each car. In other words, an arrow for a 20 mph velocity should be about twice as long as an arrow for a 10 mph velocity.

b. What is the velocity of the Pontiac relative to the Toyota?
Ask yourself this: if these two cars collided, at what speed would they crash into each other?

c. What is the velocity of the Toyota relative to the Pontiac?

d. What is the velocity of the Pontiac relative to the Pontiac?

iii. A Pontiac travels at a constant velocity of 30 miles/hour due north, relative to a Toyota. The Toyota travels at a constant velocity of 60 miles/hour due south, relative to a gas station.

a. Apply GPR 4, Rule #3 to find the velocity of the Pontiac relative to the gas station. When you do this, pay close attention to signs/directions.

b. Draw a diagram of the two cars, from the ***gas station's*** frame of reference (stationary helicopter). Include an arrow to show the velocity of the Toyota relative to the gas station and another arrow to show the velocity of the Pontiac relative to the gas station

c. Create a new diagram showing the ***Toyota's frame of reference***. Add arrows to represent the V of the Pontiac relative to the Toyota and the V of the gas station relative to the Toyota. (Use GPR 4, Rule #2 to find the Vs you need.)

d. Create a third diagram showing the ***Pontiac's frame of reference***.

B. Relative Velocities in 2-D

A Pontiac travels at 30 miles/hour at 30 degrees North-East, relative to a Toyota. The Toyota travels at 60 miles/hour at 20 degrees South-East, relative to a gas station.

In this problem, we are interested how the cars are ***moving*** (velocity), not ***where*** they are (position), so just to make it easier to picture, assume that both cars start out ***at*** the gas station, at t=0, and drive ***away*** from it.

i. What is the velocity of the Toyota relative to the Pontiac? (GPR 4, Rule #2)

ii. What is the velocity of the Toyota relative to the Toyota? (GPR 4, Rule #1)

iii. Using only ***carefully drawn*** vector diagrams, without doing any math, apply GPR 4, Rule #3, to find the approximate speed and direction of the Pontiac relative to the gas station. Follow these steps:

- **a.** In light pencil, draw a pair of coordinate axes. Label north, south, east, and west.
- **b.** Using a straight-edge, carefully draw an arrow to represent the ***Pontiac's*** velocity relative to the ***Toyota***. (Use the examples below to help you get the angle right.)
- **c.** Using a straight-edge, carefully draw an arrow to represent the ***Toyota's*** velocity relative to the ***gas station***. This arrow should begin where the arrow from step ***b*** ended. It may help to draw a horizontal line at the tip of the first arrow to help you measure out the angle.
- **d.** Draw resulting vector from the tail of the first arrow to the tip of the second. This is the velocity of the ***Pontiac*** relative to the ***gas station***.

Angle drawing references:

a 60° angle　　a 45° angle　　a 30° angle　　a 20° angle

iv. Draw a bird's-eye-view diagram, ***from the gas station's frame of reference***:

- **a.** Draw ***one dot*** to represent the starting location of ***all three objects***.
- **b.** Draw a pair of coordinate axes (East-West and North-South), which meet at the dot. (In other words, ***the three objects start at the origin***.)
- c. Based on your work in step iii above, draw an arrow to represent ***the Pontiac's velocity*** relative to (in this case, away from) ***the gas station***.
- **d.** Draw an arrow to represent the ***Toyota's*** velocity relative to the gas station.

v. Using the same process, draw two more diagrams: one showing the ***Toyota's frame of reference***, the other showing the ***Pontiac's frame of reference***.

vi. You have now drawn three diagrams, representing three different frames of reference. Which (if any) of these three perspectives is the ***most correct***? I.e. which one shows how things are *really* moving? Justify your answer. (2-3 sentences.)

II. RELATIVE ACCELERATION VECTORS

Mathematically, relative acceleration works ***just like*** relative velocity & position.
In terms of ***the laws of physics***, however, it's very different.
The difference is simply this: acceleration is ***not*** relative!

The laws of physics do ***not*** hold in an ***accelerated*** reference frame!
You ***can*** measure the ***acceleration*** of a solitary object!
Acceleration is a ***property***, ***not*** just a relation!

We can still ***talk about*** and ***calculate*** relative accelerations—the same exact way we calculate relative positions and velocities. But we ***cannot*** do physics in an accelerated reference frame. We can only do physics in an ***un***accelerated reference frame.

A Pontiac accelerates at 30 mi/hr/hr at 30 degrees North-East relative to a Toyota. A Toyota accelerates at 60 mi/hr/hr, 20 degrees South-East, relative to a gas station.

i. What is the acceleration of the Toyota relative to the Pontiac?

ii. What is the acceleration of the Toyota relative to the Toyota?

iii. What is the acceleration of the Pontiac relative to the gas station?

iv. Assume that all three objects start out at the same location and accelerate away from each other.

Draw a bird's-eye-view diagram, ***from the gas station's frame of reference***. Follow these steps:

a. Draw ***one dot*** to represent the starting location of ***all three objects***.

b. Draw a pair of coordinate axes (East-West and North-South), which meet at the dot. (In other words, ***the three objects start at the origin***.)

c. Draw an arrow to represent ***the Pontiac's acceleration*** relative to (in this case, away from) ***the gas station***.

d. Draw an arrow to represent the ***Toyota's acceleration*** relative to the gas station.

v. Using the same process, draw two more diagrams: one showing the ***Toyota's frame of reference***, the other showing the ***Pontiac's frame of reference***.

vi. You have now drawn three diagrams, representing three different frames of reference. Which (if any) of these three perspectives is the ***most correct***? I.e. which one shows how things are *really* moving? Justify your answer. (2-3 sentences.)