Putting It All Together

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Recall that $W_G = mg\Delta h$
Recall that, if an object is swinging in a circle, then $a_{radial} = a_{centripetal} = \frac{v^2}{r}$

1. Warm-Ups

A. Iron Man (mass 120 kg) takes off from the ground. He flies upwards and eastwards at an angle 30 degrees above the horizontal, until he reaches a vertical elevation of 100 meters. Compute the work done by gravity as Iron Man travels along this path.

B. Compute Iron Man's potential energy due to gravity at this moment. (You can define ground-level as zero gravitational potential energy.)

C. Next, Iron man turns and flies upwards and westwards at an angle of 45 degrees, until he reaches a vertical elevation of 200 meters.

D. Compute Iron Man's potential energy due to gravity now.

E. Suddenly, Iron Man's suit malfunctions and he falls back to the ground. Use energy and/or work to determine Iron Man's kinetic energy and speed just before he hits the ground.

2. Super Pendulum

A simple pendulum is constructed by attaching a .2 kg mass to a 1m length of string. The bob is lifted to an angle of 30 degrees and released from rest.

A. Compute the velocity of the bob when it reaches the bottom of its swing.
B. Draw an FBD of the bob at the instant that it reaches the bottom.
C. Compute the centripetal acceleration that the bob is experiencing at this moment.
D. Compute the tension on the string.
E. In 2-3 sentences, explain why the centripetal acceleration will not be the same at any other moment in the bob's swing. In your explanation, state where in the swing the centripetal acceleration will be at its ***lowest*** and its ***highest***.
F. In 2-3 sentences, explain why the tension on the string will not be the same at any other point in the bob's swing.

(problem continued next page)

The pendulum is brought back to equilibrium. It is then given an initial velocity of 9 m/s in the horizontal direction (i.e. tangent to the circle). The initial velocity is so large, that the bob manages to swing in a complete circle around the fulcrum.

G. Find the ball's speed at the moment it reaches the TOP of the circle.
H. Find the ball's centripetal acceleration at this moment.
I. Draw an FBD of the bob at this moment.
J. Compute the tension on the string.
K. In 2-3 sentences, explain why the bob goes in a complete circle, rather than falling down due to gravity, once it reaches the top half of its arc.

The pendulum is again brought back to equilibrium and the bob is given a new initial velocity.

L. Compute the initial velocity that the bob must have at the bottom, in order to make it all the way around in a complete circle.

II.G. Guide to Super Pendulum

A. Compute the velocity of the bob when it reaches the bottom of its swing.
   - It's very hard to find this with kinematics. Use work/energy instead.
   - What ONE force is doing ***work*** on the bob as it swings downwards?
   - Which way does this force point?
   - What is the ***displacement*** in this direction?
     - That last question is tricky.
     - To answer it, you'll need some careful diagrams and some trig.
     - First find the vertical displacement from the fulcrum to the bob's starting point.
     - Next find the vertical displacement from the fulcrum to the bottom of the swing.
     - If you think carefully, you'll see how to find the displacement you need.
   - What is the initial kinetic energy of the bob?
   - Compute the final kinetic energy of the bob. (W-KE Theorem)
   - Compute the final speed of the bob. (Definition of KE)

C. Compute the centripetal acceleration that the bob is experiencing at this moment.
   - Depends on your answer to A.

D. Compute the tension on the string.
   - Use your FBD from part B.
   - Apply Newton's Second Law to your radial axis.
   - There are TWO forces on this axis.

F. In 2-3 sentences, explain why the tension on the string will not be the same at any other point in the bob's swing.
   - TWO reasons: one has to do with a_{cent}, one has to do with the directions of forces.

G. Find the ball's speed at the moment it reaches the TOP of the circle.
   - Similar to A, but:
   - this time the ball's moving upwards not downwards, so work done by gravity is…
   - and this time delta-h is easier to find, because there's no trig involved.

J. Compute the tension on the string.
- Just like D.

K. In 2-3 sentences, explain why the bob goes in a complete circle, rather than falling down due to gravity, once it reaches the top half of its arc.
- Look at your FBD.
- Look at your Newton's Second Law Equation.
- Remember what it means to go in a circle: $a_c = v^2/r$
- To fall downwards would be to accelerate downwards at a rate greater than that.
- Why is tension not zero? What role is tension playing?

L. Compute $V_{BOT\text{-}MIN}$ the initial velocity that the bob must have at the ***bottom***, in order to make it all the way around in a complete circle.
- This is definitely tricky.
- The first job (which is not easy) is to find $V_{TOP\text{-}MIN}$ the minimum speed the bob can have at the TOP in order to keep spinning in a circle.
  - It will help if you made sense of part K.
  - Ask yourself: if the bob continues going in a circle, then what must be true about its acceleration on the radial axis?
  - Imagine you were swinging the bob around and around in a circle. If you kept spinning it slower and slower, what would happen to the force of tension on the string?
  - If you're not sure, think about what role T plays in the bob's motion.
  - As speed decreases, what happens to centripetal acceleration?
  - What does this do to T?
  - At the LIMIT, when you reach the MINIMUM possible value of V that will keep the bob going in a circle, T will be…………………………………… zero.
  - Apply Newton's Second Law to your radial axis.
  - Compute a_{radial}, when T is zero.
  - If the bob is still going in a circle, then a_{radial} must equal what?
  - From this you can find $V_{TOP\text{-}MIN}$.
- OK, so much for that. Now. Next step. Find the initial velocity at the bottom that will lead to $V_{TOP\text{-}MIN}$ at the top.
  - This is a lot like steps A and G.
  - Compute $K_{TOP\text{-}MIN}$, the kinetic energy of the bob if it has speed $V_{TOP\text{-}MIN}$.
  - Compute the work that gravity will do on the bob as it goes up.
  - Apply the W-KE theorem.
  - Apply the def of KE.
  - Find $V_{BOT\text{-}MIN}$.
  - You can do it!