NII.3: Dynamics & Statics

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I. Friction Warm Ups

A. Moxie, a 4kg calico cat (whose silvery hairs you may occasionally have spied clinging to Prof. Bean's suit jacket), is asleep atop a 1.5 kg laptop computer (which you also may have seen before). The computer sits on a desk. The coefficients of friction between the computer and the desk is given as $\mu_s = 0.8$ and $\mu_k = 0.7$.

i. Draw a system schema for the whole situation.
ii. Draw an FBD of Moxie and the laptop ***as one system***.
iii. Compute the normal force between the laptop and the table.
iv. Compute the maximum force of static friction between the laptop and the desk.

At some moment in time, Bean begins exerting a purely horizontal force of 50 N on the laptop.

v. Show that Prof. Bean will succeed in moving the laptop.
vi. Compute the force of kinetic friction on the laptop.
vii. Compute the acceleration of the laptop across the desk.

Suddenly, Moxie wakes up, leaps off the laptop, and goes off to do something important in the kitchen. Assume that Bean continues pushing with the same force as before.

viii. Compute the new force of kinetic friction on the laptop.
ix. Compute the new acceleration of the laptop across the desk.

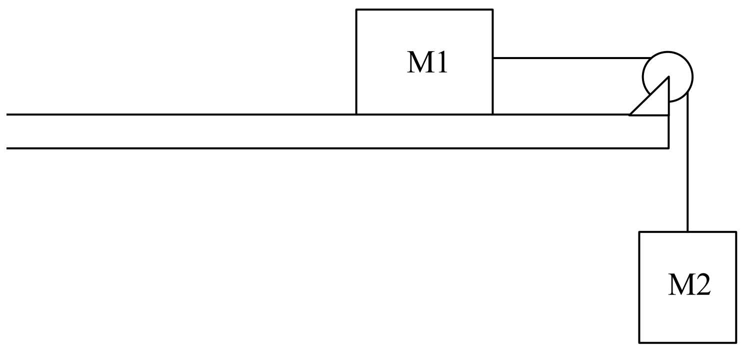
B. A 22kg sack of apples is lying on the ground. The coefficients of friction between the ground and the bag are mu_s = 0.6 and mu_k = 0.4. An old lady grabs the bag of apples and begins pulling it diagonally upwards with a force of 100N, at an angle 60 degrees above the horizontal.

i. Draw a picture of the situation a System Schema focusing on the bag of apples.
ii. Draw a pure FBD and a component FBD of the sack of apples.
iii. Compute the normal force between the sack and the ground.
Hint: set up an NII equation for the y components.
iv. Compute the maximum force of static friction on the bag of apples.
v. Show that the woman will ***not*** succeed in moving the bag.
Hint: set up an NII equation for the y components.
vi. The woman now pulls with a force of 140 N.
vii. Compute the normal force between the sack and the ground.
viii. Show that the woman ***will*** succeed in moving the bag.
ix. Compute the force of kinetic friction.
x. Compute the acceleration of the bag.

II. Double Block with String

M1 sits on a table and is attached by a string to M2, which hangs off the edge of the table, as shown. The string runs over a pulley wheel at the edge of the table. The string is massless and the pulley wheel is massless & has zero friction at its axel—in other words, it changes the direction of the string & thus of the force of tension, without absorbing any of that force. In short, the force of tension on M1 is equal & opposite to the force of tension on M2.

The coefficients of static and kinetic friction between M1 and the table are given as $\mu_k = 0.5$ and $\mu_s = 0.7$.



A. Draw two system schemata focusing on each of these blocks—or combine into one.

Assume that M1 has a mass of 10kg and M2 has a mass of 8kg. Assume further that the blocks ***are moving***. M1 is moving ***right*** and M2 is moving ***down***.

B. What force is causing M2 to move? What force is causing M1 to move?
C. If the string is not at all stretchy, what can you assume about the ***speeds*** of the two blocks.
D. Draw an FBD of M2.
E. Create an NII equation for M2. You will have two unknowns in your equation.
F. Draw an FBD of M1.
G. Create an NII equation for M1 on the y-axis.
H. Compute the normal force on M1.
I. Compute the force of friction on M1.
J. Create an NII equation for M1 on the x-axis. You will have two unknowns in your equation.
K. Your NII equations for M1 and for M2 have the same two unknowns. Solve them simultaneously (as a system of two equations) to find the acceleration of the system and the tension on the string.

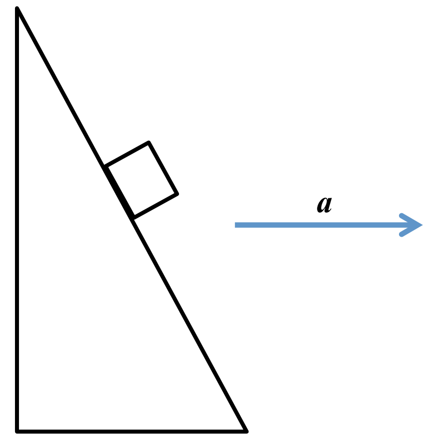
Now assume that M1 has a mass of 12kg but M2 has an unknown mass.

L. Using your FBD of M1 & M2 from step B, compute the maximum mass that M2 can have before the system will start to accelerate.

Hint: Look at the steps you performed in A-K above. The same kind of thinking, with different knowns & unknowns, can help you solve question L.

III. A Smooth Accelerating Plane

An smooth plane is inclined from the horizontal at angle of θ as shown. (Assume for now that θ = 70 degrees.) A mass ***m*** is placed at the top of the inclined plane and allowed to slide downwards (friction is negligible). At some point during the mass's trip, however, the plane begins to accelerate to the right at a rate of ***a***, causing the mass to ***stop*** sliding on the plane.



How/why does it stop sliding? That's the question. Only at one particular acceleration will the block stop sliding. Our goal is to find that acceleration.

A. Draw a picture of the situation and a system schema.
B. Draw a pure FBD of the mass.
C. Choose a coordinate system in which the x-axis is parallel to the direction of acceleration.
   Hint: This may prove ***simpler*** than you first think; if it looks simple at first, that's probably a good sign. Remember, the mass ***stopped*** sliding.
D. Create a component FBD of the mass.
E. Create NII equations for the mass in both directions.
F. Look at your component FBD & your NII equations. In 1-2 sentences, explain how the mass could have stopped sliding down the plane. What force was causing it to slide downwards before the plane started accelerating? What force (or what component of what force) balanced that force?
G. Use the NII equation in the y-direction to compute the magnitude of the Normal force, if the block is not sliding on the plane.
H. Use the NII equation in the x-direction to compute the acceleration ***a*** that the plane must be accelerating at, if the block is not sliding on the plane.

Some reflections: answer each question below in 1 sentence of English. No numbers required.

I. What will happen to the block if the acceleration drops below the ***a*** you found in step H?
J. What will happen to the block if the acceleration goes above the ***a*** you found in step H?
K. What will happen to the block if the acceleration is exactly ***a***? (Not a trick question. An "are you with me?" question.)

The GENERAL CASE: This looks scarier, but it's really just exactly what you did in steps D-H all over again, with one tiny change.

L. Now, instead of assuming that θ = 70, assume that θ is some unknown angle. Recreate the steps you did above, leaving θ as θ. So, for example, where you had sin(70) before, you'll know have sin(θ). Of course, you can't put sin(θ) into your calculator, so you'll just leave it as sin(θ). Ditto with cos(θ).

IV. A ROUGH ACCELERATING PLANE

Same situation, but now the plane is ***rough***. It has a static friction coefficient μ_s.

Of course, having static friction between the plane and the block will make it easier for the block to stay in place. Instead of just having ***one*** acceleration that will keep the block in place, there is now a ***range*** of accelerations that will do the trick. Our goal is to find this range: in other words, our goal is to find the ***maximum*** and ***minimum*** possible values of a that will keep the block in place.

Static friction can point in either direction, even though the object is not sliding.

A. If the acceleration of the plane is ***a little less than*** the $\boldsymbol{a}$ you found in part K of the previous problem (III), what direction will friction have to point to keep the block in place?
B. If the acceleration of the plane is ***greater than*** the $\boldsymbol{a}$ you found in part K of the previous problem (III), what direction will friction have to point to keep the block in place?
C. If the acceleration gets ***too*** small, what will happen to static friction and to the block?
D. If the acceleration gets ***too*** big, what will happen to static friction and to the block?

First, we'll find the ***minimum*** value that a can be to keep the block in place. To do this, we'll use the scenario from part A above. The minimum value will occur when static friction is at its maximum.

E. Draw a picture and a system schema for the situation.
F. Draw a ***pure*** FBD of the mass for this scenario (acceleration is less than the $\boldsymbol{a}$ you found in part K of the previous problem.)
G. Choose a coordinate system in which the x-axis is parallel to the direction of acceleration.
H. Draw a ***component*** FBD of the mass.
I. Write NII equations for the mass on both axes.
J. Find max static friction in terms of μ_s and **N**. (Super simple. So simple it might confuse you. Until you realize how simple it is.)
K. You are interested in the scenario where the block is ***not*** sliding on the plane. This means that one of your NII equations will have an acceleration of zero. Use this equation to find the force normal force in terms of m, g, μ_s, and (the sin and cosine of) θ.
L. Use the other equation to find **N** in terms of m, a, μ_s, and (the sin and cosine of) θ.
M. Solve to find $a_{\min}$ in terms of g, μ_s, and (the sin and cosine of) θ. All other variables should drop out.

Now we need to find the maximum value that $\boldsymbol{a}$ can be to keep the block in place. You're going to use the same process from steps F-M above (you don't have to re-do E, it's the same), except you'll use the scenario from part B (acceleration is greater than than the $\boldsymbol{a}$ you found in part K of the previous problem.)

N. Find $a_{\max}$ in terms of g, μ_s, and (the sin and cosine of) θ.

Hint: the final answer is…
$$\frac{g(\sin\theta - \mu\cos\theta)}{\cos\theta + \mu\sin\theta} \le a \le \frac{g(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}.$$

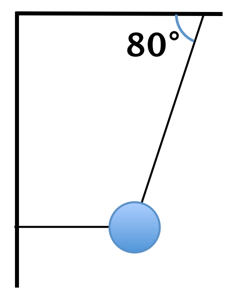
V. Elevator Review

A man stands on a scale in an elevator. The elevator moves ***downwards*** at a velocity of 10 m/s. The elevator accelerates ***upwards*** at a rate of 5 m/s^2. The scale under the man reads 180 lbs.

A. Is the elevator slowing down or speeding up?
B. Given that 1 pound = 4.5 Newtons, compute the man's mass?
   HINT: normal force is not necessarily equal to weight. You have to use NII.
C. The elevator reaches a complete stop and begins moving upwards, still moving with the same (constant) acceleration of 5 m/s^2. What will happen to the reading on the scale? Will it go upwards, downwards, or stay the same?
D. Suddenly, the elevator begins to accelerate ***downwards***. What will happen to the reading on the scale? Will it go upwards, downwards, or stay the same?
E. What downward acceleration must the elevator achieve in order for the scale to read 0?

VI. Hang Review

A 10kg metal disk is suspended by a long wire from the ceiling of a room. A second, shorter wire is attached to the disk, in order to pull it towards a nearby wall. The short wire is perfectly horizontal, and the second wire forms an 80 degree angle with the horizontal, as shown in the diagram. The disk is ***not moving***.



A. Draw a system schema for the situation.
B. Draw a ***pure*** FBD of the disk.
C. Draw a ***component*** FBD of the disk.
D. Compute the force of tension on the short wire.

   Not sure how to start? Try following these steps:

   Step 1: Create Newton II equations for each direction (x & y).
   Step 2: You have ***two*** unknowns in this problem: ***tension on long wire*** & ***tension on short wire***.
   Step 3: One of the directions (x or y) will have only ***one*** unknown force on it. Use this direction to find the ***tension on the long wire***.
   Step 4: Use the other direction to find the ***tension on the short wire***.