GUIDE to – NII.3: Dynamics & Statics

Physics 203, Profs. Max Bean & Daniel Martens Yaverbaum
John Jay College of Criminal Justice, the CUNY

I. Friction Warm Ups

A. Moxie, a 4kg calico cat (whose silvery hairs you may occasionally have spied clinging to Prof. Bean's suit jacket), is asleep atop a 1.5 kg laptop computer (which you also may have seen before). The computer sits on a desk. The coefficients of friction between the computer and the desk is given as $\mu_s = 0.8$ and $\mu_k = 0.7$.

i. Draw a picture & a system schema for the whole situation.

ii. Draw an FBD of Moxie and the laptop ***as one system***.
This will have only two arrows.

iii. Compute the normal force between the laptop and the table.
In calculating the force of gravity on the SYSTEM, you will use the mass of the SYSTEM, which is the combined mass of the objects that make up the system.

iv. Compute the maximum force of static friction between the laptop and the desk?

At some moment in time, Bean begins exerting a purely horizontal force of 50 N on the laptop.

v. Show that Prof. Bean will succeed in moving the laptop.
Assume that the system does not move, i.e. assume a=0.
Write down Newton's Second Law. Apply this law to your x-axis.
Solve for the force of friction.
Is the number that you found for friction possible? Why/why not?
If it is NOT possible, then you know that the assumption that a=0 was wrong

vi. Compute the force of kinetic friction on the laptop.
Because the system IS sliding, we now have to use kinetic friction.

vii. Compute the acceleration of the laptop across the desk.
Again, write down Newton's Second Law and apply it to your x-axis.
Solve for a.

Suddenly, Moxie wakes up, leaps off the laptop, and goes off to do something important in the kitchen. Assume that Bean continues pushing with the same force as before.

What changes when Moxie gets off the laptop? The mass of the system changes.
What else therefore changes?
TWO forces change: mg, of course, and therefore which other force?

viii. Compute the new force of kinetic friction on the laptop.
(i.e. redo steps iii and iv with this new scenario)

ix. Compute the new acceleration of the laptop across the desk.

B. A 22kg sack of apples is lying on the ground. The coefficients of friction between the ground and the bag are mu_s = 0.6 and mu_k = 0.4. An old lady grabs the bag of apples and begins pulling it diagonally upwards with a force of 100N, at an angle 60 degrees above the horizontal.

i. Draw a picture of the situation a System Schema focusing on the bag of apples.
ii. Draw a pure FBD and a component FBD of the sack of apples.

If the sack moves, it will presumably slide along the ground. Therefore, the direction of acceleration will be purely horizontal. This tells you that your X-axis will be horizontal (as opposed to some kind of weird tilted X-axis).

Your PFBD should have FOUR arrows. One of them will be diagonal to your axes. Split this diagonal force into components using everyone's favorite kind of triangle. Then draw your CFBD. It will have FIVE arrows.

iii. Compute the normal force between the sack and the ground.
Hint: set up an NII equation for the y components.

Don't assume that N = mg, because it DOESN'T!
There will be THREE forces on the y-axis.

iv. Compute the maximum force of static friction on the bag of apples.
v. Show that the woman will ***not*** succeed in moving the bag.
Hint: set up an NII equation for the y components.

Assume that the system does not move, i.e. assume a=0.
Write down Newton's Second Law. Apply this law to your x-axis.
Solve for the force of friction.
Is the magnitude that you found for friction possible? Why/why not?
If it IS possible, then your assumption was correct, and a=0.

vi. The woman now pulls with a force of 140 N. Not actually a question!
vii. Compute the normal force between the sack and the ground.

See advice for step iii above.

viii. Show that the woman ***will*** succeed in moving the bag.

Notice that the Normal force with the ground ***has changed***.
Therefore f_s(max) has changed.
Compute the new maximum value of static friction.
Then follow the advice for step v above.

ix. Compute the force of kinetic friction.

Because the system IS sliding, we now have to use kinetic friction.

x. Compute the acceleration of the bag.

Again, write down Newton's Second Law and apply it to your x-axis. Solve for a.

II. Double Block with String

M1 sits on a table and is attached by a string to M2, which hangs off the edge of the table, as shown. The string runs over a pulley wheel at the edge of the table. The string is massless and the pulley wheel is massless & has zero friction at its axel—in other words, it changes the direction of the string & thus of the force of tension, without absorbing any of that force. In short, the force of tension on M1 is equal & opposite to the force of tension on M2.

The coefficients of static and kinetic friction between M1 and the table are given as $\mu_k = 0.5$ and $\mu_s = 0.7$.

A. Draw two system schemata focusing on each of these blocks—or combine into one.

Assume that M1 has a mass of 10kg and M2 has a mass of 8kg. Assume further that the blocks ***are moving***. M1 is moving ***right*** and M2 is moving ***down***.

B & C not shown in this document.

D. Draw an FBD of M2. TWO arrows.

E. Create an NII equation for M2. You will have two unknowns in your equation.

This equation contains a, m, mg, and T. Which two ***don't*** you know?

F. Draw an FBD of M1. FOUR arrows.

G. Create an NII equation for M1 on the y-axis. 2 forces. M1 is NOT moving up or down.

H. Compute the normal force on M1. No tricks here.

I & J not shown in this document.

K. Your NII equations for M1 and for M2 have the same two unknowns. Solve them simultaneously (as a system of two equations) to find the acceleration of the system and the tension on the string.

If you read carefully, we just gave away what the two unknown variables are. Depending on how you set up your coordinate system $a_{M1} = a_{M2}$ or $a_{M1} = -a_{M2}$

Now assume that M1 has a mass of 12kg but M2 has an unknown mass.

L. Using your FBD of M1 & M2 from step B, compute the maximum mass that M2 can have before the system will start to accelerate.

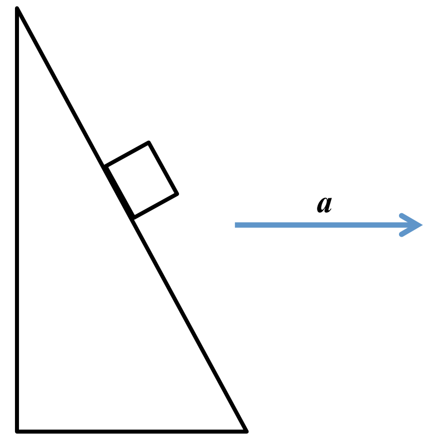
Hint: Look at the steps you performed in A-K above. The same kind of thinking, with different knowns & unknowns, can help you solve question L.

If you put bigger & bigger masses on M2, the system will eventually start to accelerate. You want the maximum mass that you can put so it WON'T accelerate, but if you made the mass even a tiny bit larger, it would start to accelerate.

As you increase the mass on M2, friction will have to work harder and harder to keep the system from moving. What is the strongest friction can get? What mass will M2 have when friction reaches its maximum?

III. A Smooth Accelerating Plane

An smooth plane is inclined from the horizontal at angle of θ as shown. (Assume for now that θ = 70 degrees.) A mass ***m*** is placed at the top of the inclined plane and allowed to slide downwards (friction is negligible). At some point during the mass's trip, however, the plane begins to accelerate to the right at a rate of ***a***, causing the mass to ***stop*** sliding on the plane.



How/why does it stop sliding? That's the question. Only at one particular acceleration will the block stop sliding. Our goal is to find that acceleration.

A. Draw a picture of the situation and a system schema.
B. Draw a pure FBD of the mass. TWO arrows.
C. Choose a coordinate system in which the x-axis is parallel to the direction of acceleration.
   Hint: This may prove ***simpler*** than you first think; if it looks simple at first, that's probably a good sign. Remember, the mass ***stopped*** sliding.
   …the mass stopped sliding. In other words, the mass and the plane are moving ***together***.

D. Create a component FBD of the mass.
   You will have to split **N** into components. You don't yet know **N**, so you will leave these components in terms of **N**. They might be things like (0.44)N—but not that exact number. You should end up with three arrows.

E. Create NII equations for the mass in both directions.
   i.e. one for the x-axis and one for the y-axis.
   The x-axis equation will have ONE force. The y-axis equation will have TWO.

F. Look at your component FBD & your NII equations. In 1-2 sentences, explain how the mass could have stopped sliding down the plane. What force was causing it to slide downwards before the plane started accelerating? What force (or what component of what force) balanced that force?
   This part is tricky. Before answering, ask yourself: is the mass accelerating at all? If so, in what direction is it accelerating?

   When you think about this question, remember that (UNLIKE velocity), acceleration is NOT relative. The block might not be accelerating relative to the plane, but that does NOT mean it's not accelerating.

   You do not want to think in the plane's frame of reference, because the plane's frame of reference is an ACCELERATED frame of reference.

   Once you see which way the block is accelerating, ask yourself: which way is the block NOT accelerating? Why not? What two force components are balanced?

G. Use the NII equation in the y-direction to compute the magnitude of the Normal force, if the block is not sliding on the plane.

If you understood step F above, this should be a sinch.
Or, another way to think about it, if the block is not sliding, is it moving up/down?
What is a_y?
The NII equation has two forces. You know one of them. You know a_y.
Leave ***m*** as a variable.
Solve for **N** in terms of the variable ***m***.

H. Use the NII equation in the x-direction to compute the acceleration ***a*** that the plane must be accelerating at, if the block is not sliding on the plane.

a_x is not zero. The block is not accelerating ***relative to the plane***, but it is accelerating.
We don't yet know how much. Therefore, leave a_x as a variable.
There is only one force on this axis.
Solve for a_x. The variable ***m*** will cancel out, and you will get a final NUMBER for a_x.

Some reflections: answer each question below in 1 sentence of English. No numbers required.

I. What will happen to the block if the acceleration drops below the ***a*** you found in step H?
Go back to step H: if a_x got smaller, what would happen to **N**?
Now go back to step G. If N changes but mg stays the same, what will happen to a_y?
What does this mean? What will the block do?

J. What will happen to the block if the acceleration goes above the ***a*** you found in step H?
Go back to step H: if a_x got bigger, what would happen to **N**?
Now go back to step G. If N changes but mg stays the same, what will happen to a_y?
What does this mean? What will the block do?

K. What will happen to the block if the acceleration is exactly ***a***? (Not a trick question. An "are you with me?" question.)
If you understand I and J, this should be a sinch.

The GENERAL CASE: This looks scarier, but it's really just exactly what you did in steps D-H all over again, with one tiny change.

L. Now, instead of assuming that $\boldsymbol{\theta}$ = 70, assume that $\boldsymbol{\theta}$ is some unknown angle. Recreate the steps you did above, leaving $\boldsymbol{\theta}$ as $\boldsymbol{\theta}$. So, for example, where you had sin(70) before, you'll know have sin($\boldsymbol{\theta}$). Of course, you can't put sin($\boldsymbol{\theta}$) into your calculator, so you'll just leave it as sin($\boldsymbol{\theta}$). Ditto with cos($\boldsymbol{\theta}$).

EXACTLY the same steps, just with variables instead of numbers.
Your final answer will be in terms of g and the sin and cosine of $\boldsymbol{\theta}$.

IV. A ROUGH Accelerating Plane

Same situation, but now the plane is ***rough***. It has a static friction coefficient μ_s.

Of course, having static friction between the plane and the block will make it ***easier*** for the block to stay in place. Instead of just having ***one*** acceleration that will keep the block in place, there is now a ***range*** of accelerations that will do the trick. Our goal is to find this range: in other words, find the ***maximum*** and ***minimum*** possible values of a that will keep the block in place.

Static friction can point in either direction, even though the object is not sliding.

A, B, C & D not shown in this document.
When thinking about A-D, think back to your answers to parts I and J of problem III.

First, we'll find the ***minimum*** value that a can be to keep the block in place. To do this, we'll use the scenario from part A above. The minimum value will occur when static friction is at its maximum.

E. Draw a picture and a system schema for the situation.
F. Draw a ***pure*** FBD of the mass for this scenario (acceleration is less than the a you found in part K of the previous problem.) THREE arrows.
G. Choose a coordinate system in which the x-axis is parallel to the direction of acceleration. Remember, the block is NOT sliding on the plane, but the plane IS accelerating.
H. Draw a ***component*** FBD of the mass. FIVE arrows.
I. Write NII equations for the mass on both axes.
J. Find max static friction in terms of μ_s and **N**. (Super simple. So simple it might confuse you. Until you realize how simple it is.)
You need f_s(max) IN TERMS of μ_s and **N**. Yes, it is THAT simple.
K. You are interested in the scenario where the block is ***not*** sliding on the plane. This means that one of your NII equations will have an acceleration of zero. Use this equation to find the force normal force in terms of m, g, μ_s, and (the sin and cosine of) θ.
It's just algebra, folks.
L. Use the other equation to find **N** in terms of m, a, μ_s, and (the sin and cosine of) θ.
M. Solve to find $a_{\min}$ in terms of g, μ_s, and (the sin and cosine of) θ. All other variables should drop out.
You have TWO equations in terms of N. Set them equal and solve for a.

Now we need to find the maximum value that a can be to keep the block in place. You're going to use the same process from steps F-M above (you don't have to re-do E, it's the same), except you'll use the scenario from part B (acceleration is ***greater*** than the a you found in problem III.)

N. Find $a_{\max}$ in terms of g, μ_s, and (the sin and cosine of) θ.

Hint: the final answer is…
$$\frac{g(\sin\theta - \mu\cos\theta)}{\cos\theta + \mu\sin\theta} \le a \le \frac{g(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}.$$

Problems V & VI not shown in this document