# Lab 2: A Thing on a String
# (The Planar Pendulum)

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*** NOTE: Every Triple-Starred Direction (***) is a direction in response to which you must write something down. Every lab write-up requires at least your written responses to all ***'d directions. (Some write-ups may require more. Always check the web to see if there is a particular set of supplementary lab write-up instructions due for a given lab period.)

## A1. **Ultimate Goals**:

1. To become re-acquainted and pre-acquainted with the periodic motion of a simple, planar pendulum.
2. To build a generalized understanding of "Simple Harmonic Motion" from this example.

## A2. **Specific Goals**:

1. To build experimental evidence for the claim that small-angle swings of a planar pendulum are approximately simple harmonic.
2. To build an analytical justification for the claim that small-angle swings of a planar pendulum are approximately simple harmonic.
3. To use the simple harmonic nature of a pendulum to verify the accepted value for free-fall acceleration near the surface of the Earth.

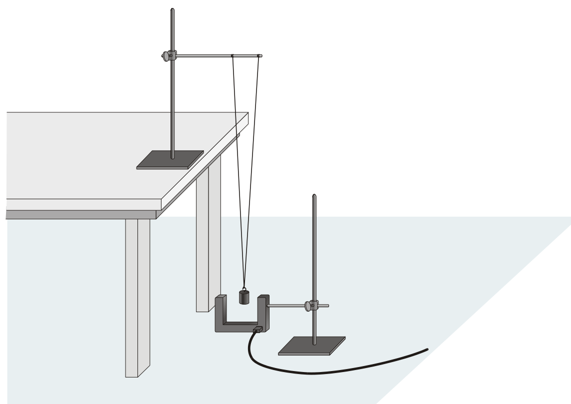
## B1. **Necessary Equipment**

1. Ringstand and/or Stand with Clamp
2. Light, Strong String
3. Various Masses: Ideally, 100 g, 200 g and 300 g
4. Stopwatch
5. Meter Stick
6. Balance (or factory-based knowledge of masses)
7. Laptop Computer
8. Vernier Computer Interface
9. Vernier Photogate
10. Logger ***Pro***
11. Vernier Photogate
12. Protractor

B2. **Necessary Mathematical Relations**

1. $\theta \equiv \frac{x}{R}$, **when measured in RADIANS.**

2. $\lim_{\theta \to 0} \sin\theta = \theta$, **when measured in RADIANS.**



C. **Procedures**

1. Use the ring stand to hang the 200 g mass from two strings. Attach the strings to a horizontal rod about 10 cm apart, as shown in Figure 1. This arrangement will let the mass swing only along a line, and will prevent the mass from striking the Photogate. The length of the pendulum is the distance from the point on the rod halfway between the strings to the center of the mass. The pendulum length should be at least 1 m.

2. Attach the Photogate to the second ring stand. Position it so that the mass blocks the Photogate while hanging straight down. Connect the Photogate to DIG/SONIC 1 on the interface.

3. Open the file "14 Pendulum Periods" in the *Physics with Vernier* folder. A graph of period *vs*. time is displayed.

4. Temporarily move the mass out of the center of the Photogate. Notice the reading in the status bar of Logger *Pro* at the bottom of the screen, which shows when the Photogate is blocked. Block the Photogate with your hand; note that the Photogate is shown as **blocked**. Remove your hand, and the display should change to **unblocked**. Click ▶ Collect and move your hand through the Photogate repeatedly. After the first blocking, Logger *Pro* reports the time interval between every other block as the period. Verify that this is so.

5. Now you can perform a trial measurement of the period of your pendulum. Pull the mass to the side about 10º from vertical and release. Click ▶ Collect and measure the period for five complete swings. Click ■ Stop. Click the Statistics button, STAT, to calculate the average period. You will use this technique to measure the period under a variety of conditions.

**Part I Amplitude**

6. Determine how the period depends on amplitude. Measure the period for five different amplitudes. *As always, that means: Do 3-5 repeated trials for each possible value for the independent variable and average the results*—that will entail a total of 15 – 25 swings. Use this same approach toward reducing random error in all subsequent parts of this lab.

   Use a range of amplitudes, from just barely enough to unblock the Photogate, to about 25º. Each time, measure the amplitude using the protractor so that the mass with the string is released at a known angle. Repeat Step 5 for each different amplitude.

   *** Record the data in your data table.

7. For ***every measurement*** you make in every step of ***every physics experiment***, include an estimated value for percentage of ***"Uncertainty"***:

   The ***minimum uncertainty percentage*** systematically associated with any measurement is ONE-HALF of the smallest used measurement interval DIVIDED BY the total measurement. Make certain you understand what this means and why before proceeding.

   *** Record uncertainties along with all your measurements in the data table.

**Part II Length**

8. Use the method you learned above to investigate the effect of changing pendulum length on the period. Use the 200 g mass and a consistent amplitude of 20º for each trial. Vary the pendulum length in steps of 10 cm, from 1.0 m to 0.50 m. If you have room, continue to a longer length (up to 2 m). Repeat Step 5 for each length.

   *** Record the data in the second data table below. Measure the pendulum length from the rod to the middle of the mass.

**Part III Mass**

9. Use the three masses to determine if the period is affected by changing the mass. Measure the period of the pendulum constructed with each mass, taking care to keep the distance from the ring stand rod to the center of the mass the same each time, as well as keeping the amplitude the same. Repeat Step 5 for each mass, using an amplitude of about 20°. Record the data in your data table

## DATA TABLES

### Part I Amplitude

| Amplitude (°) | Amplitde Uncertainty % | Average period (s) | Period Uncertainty % |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

### Part II Length

| Length (m) | Length Uncertainty % | Average period (s) | Period Uncertainty % |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

### Part III Mass

| Mass (kg) | Mass Uncertainty % | Average period (s) | Period Uncertainty % |
|---|---|---|---|
| | | | |
| | | | |
| | | | |

## D. Analysis

1. **EFFECTS:**

   *** Based on the pattern of observations and measurements you made, can you conjecture whether or not the oscillation of your pendulum is approximately ***Simple Harmonic***?

   For this question, answer in approximately 3 complete sentences of English. Your answer should make reference to a fundamental effect we associate with Simple Harmonic Oscillation. That is, oscillators move in a periodic fashion. As long as the oscillation is simple harmonic, we will tend to observe the period to depend on certain factors, but not on others. What kind of dependence did you observe/measure here, and what kind of independences did you observe/measure? What do these effects lead you to conclude about the oscillation?

2. **CAUSES:**

   Now, let's look behind the motion itself and endeavor to figure out ***WHY*** the oscillations tend to fall into whatever pattern they do. Recall: the cause for any change in motion is the sum of ***FORCES***.

THE ultimate GOAL of everything that you are about to do is:
Derive a 2nd order DIFFERENTIAL EQUATION for which:
THETA is the DEPENDENT VARIABLE,
TIME is the INDEPENDENT VARIABLE,
And all other terms are given constants.

Each step below is provided in order to hint/help you along this derivation.
Therefore, each step is meant to explain something that you would apply or do to a mathematical statement that you obtained in the step before.
If you think you can get to the intended result through some different steps or with different thoughts, please feel free.

a. ******* Draw a "Pure" Free-Body-Diagram of the pendulum bob at one arbitrary point in its swing—displaced from the vertical by an angle Theta.

b. *** Since the pendulum bob swings in the arc of a ***CIRCLE,*** analyze these forces through a coordinate system most convenient for circles: Break up any and all off-axis vectors into components so that every force component lies along either a Radial (Centripetal) Axis, a Tangential Axis or a "Z" Axis ("Z" being perpendicular to the plane of the circle). Draw a "Component" Free-Body Diagram according to this coordinate system.

c. *** Write down Newton's 2nd Law and apply to each axis separately.

d. *** Focus on your Newton's Law statement for the axis along which MOTION (VELOCITY) occurs: The tangential axis. Recognize that the acceleration along this axis here is not yet "known", but is, by definition, the second derivative of position with respect to time. Make this substitution in your Newton's Law statement. Note that along this axis, positions advance (change) along the arc length of the circle. Assume that the direction AWAY from the vertical is reckoned POSITIVE. Anything pointing TOWARD the vertical should get a negative sign.

e. *** Recall the definition of any angle MEASURED IN RADIANS is the ratio of arc length to radius (B2). Using this definition, re-write your Newton's Law statement so that each side of the equality is a function of nothing more than time, the angle theta, the string length and the acceleration due to gravity.

f. *** Recall that as long as we MEASURE IN RADIANS, the sine of an angle approaches the angle itself as the angle gets closer and closer to 0. Assume that the angles under discussion and observation are extremely small, re-write your Newton's 2nd Law statement as an approximate equality for small angles—one that makes reference only to angles themselves, no trigonometric functions.

g. *** You now should have a second-order differential equation for which time is the independent variable, angle is the dependent variable and the two constants are length and free-fall acceleration. Clearly write down this differential equation.

h. *** In at least three three complete sentences of English, explain why this differential equation demands that the pendulum motion SHOULD approximate simple harmonic oscillation under certain conditions. Under what condition will the oscillation start to drift away from simple harmonic?

i. *** Use your differential equation to derive :

   i. The angular frequency of the pendulum (in terms of ***l*** and ***g***),
   ii. The standard frequency of the pendulum (in terms of ***l*** and ***g),***
   iii. The PERIOD of the pendulum (in terms of ***l*** and ***g***).

j. *** PRODUCE A GRAPH of Period (***T***) as a function of Amplitude (Angle).

k. *** Using ***l*** as the independent variable and ***T*** as the dependent variable, PRODUCE TWO SEPARATE GRAPHS just like you did in Lab #1: One will be a graph of the relationship itself. The other will be a ***linearized*** version of this relationship.

l. *** Just like you did in Lab #1, use the SLOPE of your linearized graph and the relation for PERIOD that you derived in (i, iii), above, to approximate or verify a value for ***g***, the value for free-fall acceleration near the surface of the Earth.

m. *** Compare your value for g with the commonly accepted value. How does the percentage by which the two values differ compare with the uncertainty percentages you entered in the data table. Provide approximately three complete sentences of English AND quantitative analysis explaining how the measurement uncertainties might reasonably lead to the uncertainty in your final value.